Guide
to Geodetic and Vertical Datums in Quebec

June 2010

Ministère des Ressources naturelles et de la Faune

Produced by

Ministère des Ressources naturelles et de la Faune
Direction de la référence géodésique
Yves Thériault, MSc
5700, 4e Avenue Ouest, bureau E-304
Québec (Québec) G1H 6R1
Telephone: 418 627-6281
Fax: 418 646-9424
E-mail: information.geographique@mrnf.gouv.qc.ca
Internet site: www.mrnf.gouv.qc.ca

Distribution

This publication has been designed for double-sided printing.

Legal deposit – Bibliothèque et Archives nationales du Québec, 2013
ISBN 978-2-550-68133-5 (pdf)

Note to Readers

The measuring techniques and knowledge in geodetic sciences are evolving at an increasingly rapid pace and are having an influence on each other. As a result, some parts of this guide may eventually become outdated. This document presents the state of current geodetic and vertical datums in Quebec in April 2010.

The degree of detail in this document varies depending on need. The presentation and description of the different topics covered attempt, wherever possible, to remain general. Sometimes however, certain concepts must be introduced. For instance, an explanation of the role of equipotential surfaces in the vertical datum is necessary to ensure proper understanding of the inaccuracies of CGVD28.

This guide does not provide a detailed description of any measuring or calculation techniques used to determine geodetic points or bench marks. Readers who wish to learn more about this can consult some excellent references and Internet sites. The references used to produce this document can be used as a starting point.

Table of Contents

List of Tables

List of Figures

List of Abbreviations and Acronyms

AUSPOS	Online GPS Processing Service (Australia)
BIH	International Time Bureau
BTS84	BIH Terrestrial System of 1984
CACS	Canadian Active Control System
CBN	Canadian Base Network
CGD	Canadian Geodetic Datum
CGG2000	Canadian Gravimetric Geoid of 2000
CGG2005	Canadian Gravimetric Geoid of 2005
CGQ77	Compensation géodésique du Québec de 1977
CGVD28	Canadian Geodetic Vertical Datum of 1928
CHRS	Canadian Height Reference System
CORS	Continuously Operating Reference Station (US)
CORS96	Version of NAD83 based on the CORS network
CSRS	Canadian Spatial Reference System
DMA	Defence Mapping Agency (US)
DORIS	Doppler Orbitography and Radiopositioning Integrated by Satellite
DRG	Direction de la référence géodésique
ED50	European Datum of 1950
EGM96	Earth Gravitational Model of 1996
ETRF89	European Terrestrial Reference Frame of 1989
ETRS89	European Terrestrial Reference System of 1989
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
GPS-H	Height conversion software
GRACE	Gravity Recovery and Climate Experiment
GRS80	Geodetic Reference System of 1980

GSD	Geodetic Survey Division (Natural Resources Canada)
HT2.0	Height Transformation, version 2.0
IAU	International Astronomical Union
ICE-4G	Model of post-glacial rebound
IDS	International DORIS Service
IERS	International Earth Rotation and Reference Systems Service
IGLD	International Great Lakes Datum
IGS	International GNSS Service
IGS05	Reference frame of 2005 (IGS)
ILRS	International Laser Ranging Service
ITRF	International Terrestrial Reference Frame
ITRF2000	International Terrestrial Reference Frame of 2000
ITRF2005	International Terrestrial Reference Frame of 2005
ITRF2008	International Terrestrial Reference Frame of 2008
ITRF89	International Terrestrial Reference Frame of 1989
ITRF90	International Terrestrial Reference Frame of 1990
ITRF96	International Terrestrial Reference Frame of 1996
ITRF97	International Terrestrial Reference Frame of 1997
ITRS	International Terrestrial Reference System
IUGG	International Union of Geodesy and Geophysics
IVS	International VLBI Service
LLR	Lunar Laser Ranging
MSL	Mean Sea Level
MSL29	Mean Sea Level of 1929
NAD27	North American Datum of 1927
NAD83	North American Datum of 1983
NAVD88	North American Vertical Datum of 1988
NGA	National Geospatial-Intelligence Agency (US)
NGS	National Geodetic Survey (US)

NGVD29	National Geodetic Vertical Datum of 1929
NNR-NUVEL-1A	No-net-rotation, Northwestern University Velocity, model 1, modification A
NOAA	National Oceanic and Atmospheric Administration (US)
NSRS	National Spatial Reference System (US)
NSWC	Naval Surface Warfare Center (US)
OPUS	Online Positioning User Service (US)
PPP	Precise Point Positioning (Natural Resources Canada)
RINEX	Receiver Independent Exchange Format
SCOUT	Scripps Coordinate Update Tool (US)
SLD29	Sea Level Datum of 1929
SLR	Satellite Laser Ranging
SOPAC	Scripps Orbit and Permanent Array Center (US)
SST	Sea Surface Topography
TRANSIT	Navigation satellite system
TRNOBS	Coordinate Transformation software
VLBI	Very Long Baseline Interferometry
WGS72	World Geodetic System of 1972
WGS84	World Geodetic System of 1984

1 Introduction

The Earth is a complex structure that is subject to different internal and external forces that are constantly altering its structure. As a science that studies the dimensions and shape of the Earth and the variations in its gravitational field in relation to time, geodesy measures the structural and gravitational variations that affect our planet. In order to do this, it implements geodetic and vertical datums, or bench mark systems, that can situate an event or object in space and time.

With the launch of Sputnik in 1957, geodesy and its datums entered the space age. Before this, geodetic datums, which consisted primarily of measurements of angles and distances, were purely planimetric and generally regional. Today, geodetic datums are realized from space-based measurements and can be three- or four-dimensional, international or regional. Whether they are traditional (angle and distance measurements) or modern (space-based measurements), the datums use different coordinate systems to locate an object mathematically.

A “one-dimensional” datum also exists to determine the height of an object on the Earth. Unlike a geodetic datum, this vertical datum is not a mathematical surface, but a surface resulting from the gravitational field, or the geoid. This specific surface is a result of the Earth’s physics and is an approximation of mean sea level at rest and its extension over the continents.

Modern geodesy introduces new terms for the geodetic and vertical datums. Today, the language is all about “reference systems” and “reference frames.” A reference system is a theoretical concept that is defined by a set of parameters and conventions. The reference frame is the concrete realization of a reference system. For geodetic datums, there are terrestrial reference systems (also known as “geodetic reference systems”) and terrestrial reference frames. For vertical datums, there are height reference systems and height reference frames. These concepts are explained in this guide, which provides an introduction to modern datums and a description of the geodetic and vertical datums currently in use in Quebec.

The guide starts by introducing a few basic concepts to help understand the topics covered later. Before getting too deep into the topic of reference systems, it is necessary to understand the phenomena that change our planet (Chapter 2). They have differing degrees of influence on the realization of these systems. A short presentation of the coordinate systems used is given so that the terms are clearly understood (Chapter 3). Chapter 4 gives a brief overview of the conventional reference systems established before the space age, including NAD27. In Chapter 5, a description of the main international terrestrial reference systems (ITRS and WGS84) provides a basis for presenting the terrestrial reference system in use in Quebec, which is NAD83, and its two realizations. The relationships between these reference systems and their influence on GPS observations conclude this chapter. Before describing the CGVD28 height reference system and its realization, Chapter 6 starts with the role of equipotential surfaces and the various types of heights in a vertical datum. Examples of height calculations help to better understand the limitations of CGVD28 in the era of GPS. Chapter 7 concludes with future terrestrial and height reference systems.

2 The Earth

The Earth is a complex plastic (flexible) body made up of solid, semi-rigid and liquid parts that are subject to different internal and external forces. The different parts are grouped to form three main layers that constitute the Earth (Figure 1) (Bourque, 2010 [translation]): crust, mantle and core.

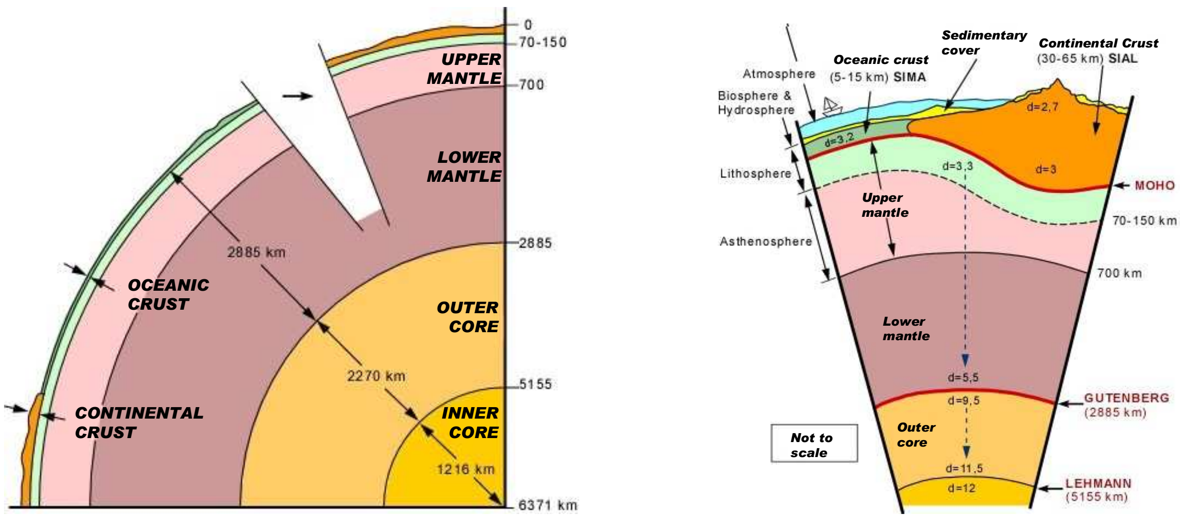


Figure 1: Structure of the Earth

The crust is the rigid layer on which we live. It is very thin compared to the size of the planet, which has a radius of about 6,371 km. At an average thickness of about 30 km, it can be 5 km thick under the oceans and 65 km thick under mountainous areas. The mantle is a mix of semi-rigid layers that increase in density towards the centre for a thickness of about 2,800 km. The core, made up of a liquid part that is denser than the mantle for 2,270 km (outer core) and a solid part for 1,220 km (inner core), is the centre of the planet.

Internal and external forces, among other things, change the Earth's shape, its rotational speed and its axis of rotation. Figure 2 (GSD, 2006) illustrates the main forces. Changes in the Earth's shape can be described according to horizontal and vertical motion that occurs in the crust.

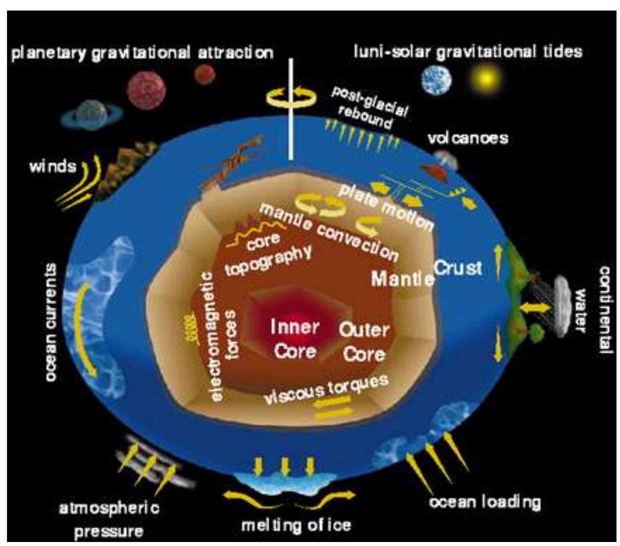


Figure 2: Influential forces

2.1 Horizontal Motion

The main motion is a result of plate tectonics. The theory of plate tectonics, called “continental drift,” was presented for the first time in 1915 by Alfred Wegener, a German. The theory, considered outlandish by his contemporaries, started to be taken seriously in the mid-20th century. Today, space-based techniques allow precise tracking of plate motion.

A lithospheric, or tectonic, plate consists of the crust and the solid part of the mantle adjacent to the crust (Figure 1). Convection and motion phenomena in the mantle material cause the lithospheric plates to move, or drift, a few centimetres per year (Figure 3) (JPL, 2006).

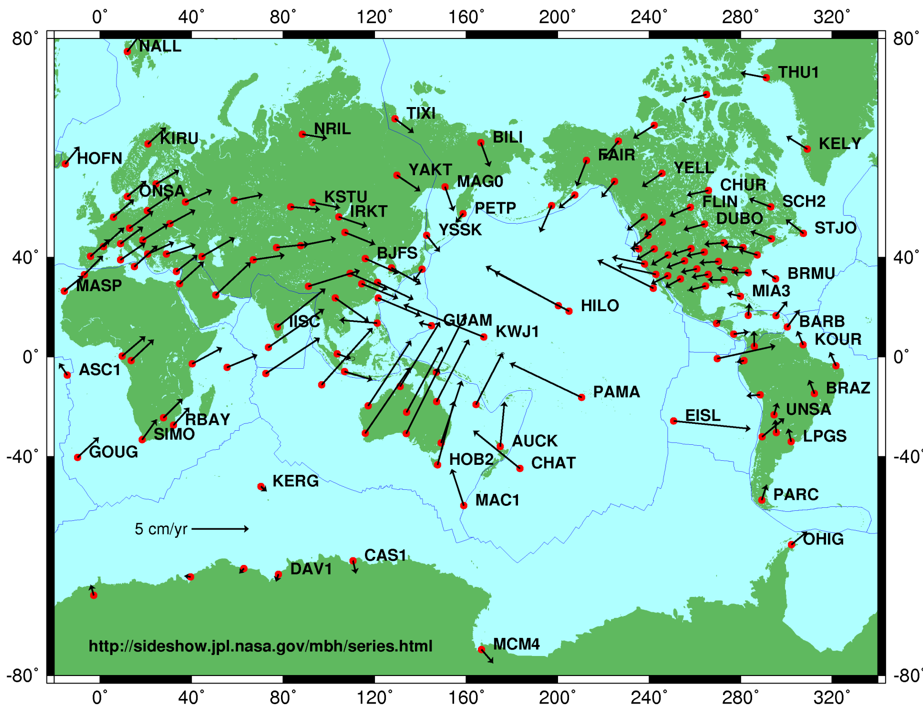


Figure 3: Main tectonic plates

There is also local motion in the order of a millimetre per year. In general, this motion comes from fault-type intraplate deformations. The forces generated by the compression from adjacent plates also cause horizontal motion.

2.2 Vertical Motion

Vertical motion is caused by regional or local phenomena, and not global events like tectonic plate drift. Such regional events include post-glacial rebound caused by the uplift of the Earth's crust. During the last glacial period about 18,000 years ago, a thick layer of ice, up to 3 km thick in some areas, covered the north of our continent. This mass compressed the crust and part of the mantle moved around the edges, creating a peripheral fore-bulge. After deglaciation, the crust is slowly returning to its shape; this is isostatic equilibrium (Figure 4) (Henton et al., 2006). As the mantle returns, it flows from the periphery back to the subduction zone. The result is a peripheral subsidence and an uplift where the mass of ice was.

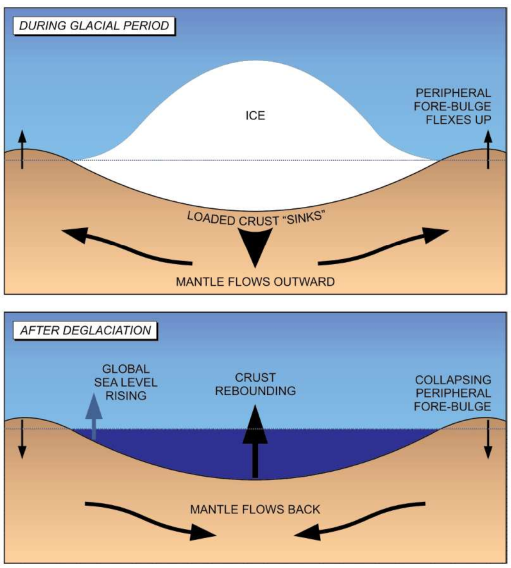


Figure 4: Isostatic equilibrium

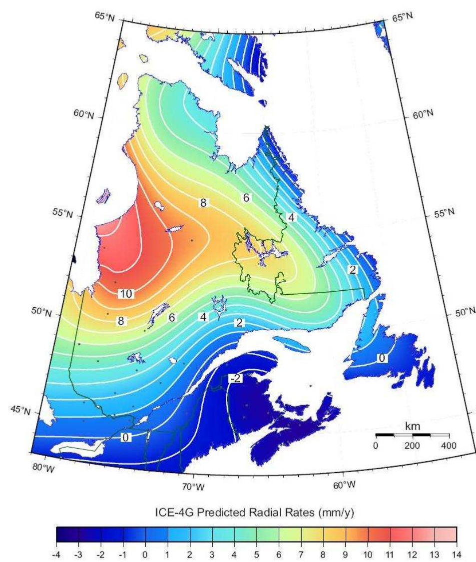
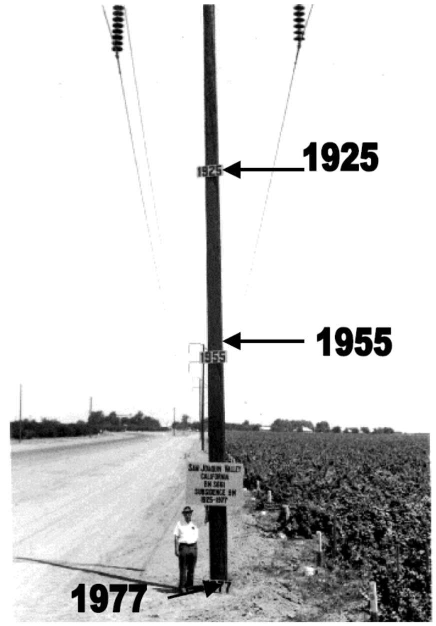


Figure 5: Post-glacial rebound

In Canada, uplift can be more than 10 mm/yr in the Hudson's Bay area, and subsidence can be several millimetres per year in the Maritime Provinces. Figure 5 (Henton et al., 2004) shows the effects of post-glacial rebound in eastern Canada based on the ICE-4G model developed in the mid-1990s. Around the maximum decompression zone, there is a horizontal motion that can reach several millimetres per year. The hinge zone around the uplift and subsidence is somewhere along or south of the St. Lawrence River depending on the different analyses done on post-glacial rebound.

Another cause of vertical motion is subsidence resulting from subsurface drying of the land or sedimentary compaction. The subsidence rate can vary from several millimetres to several centimetres per year. Figure 6 (NGS, 1998) clearly shows the effect of an annual drop of several centimetres per year caused by the pumping of ground water in California. In the long term, such a drop can have an effect on the direction of water flow.

The interaction at the boundaries of two tectonic plates also creates motion. For instance, subduction, the process in which a dense plate is forced below a less dense plate, produces uplift. This interaction also generates horizontal motion if the uplift is accompanied by a compression or tilting of the plate.



Approximate levels of subsidence. The signs show the position of land surface in 1925, 1955, and 1977. Although the rate of subsidence has decreased, the continued pumping of ground water has resulted in additional subsidence in the past 20 years.

Figure 6: Subsidence in California

2.3 Global Motion

Figures 7, 8, 9 and 10 give a graphic representation of horizontal (latitude and longitude) and vertical (height) motion for Baie-Comeau, Kuujjuarapik, Schefferville and Val-d'Or (JPL, 2008), respectively. The data come from the GPS tracking stations. The velocity of these sites is summarized in Table 1. The "Horizontal" column represents the velocity of the North American plate at our latitudes, which is about 2 cm/yr. The direction of motion at Schefferville (SCH2) is shown in Figure 3. The velocities in the "Height" column are representative of the post-glacial rebound effect on the vertical component.

Table 1: Velocity

City	Latitude (mm/yr)	Longitude (mm/yr)	Horizontal (mm/yr)	Height (mm/yr)
Baie-Comeau	5.9	-15.6	16.7	1.3
Kuujjuarapik	1.3	-17.5	17.5	10.6
Schefferville	7.5	-17.7	19.2	10.6
Val-d'Or	2.2	-17.4	17.6	9.1

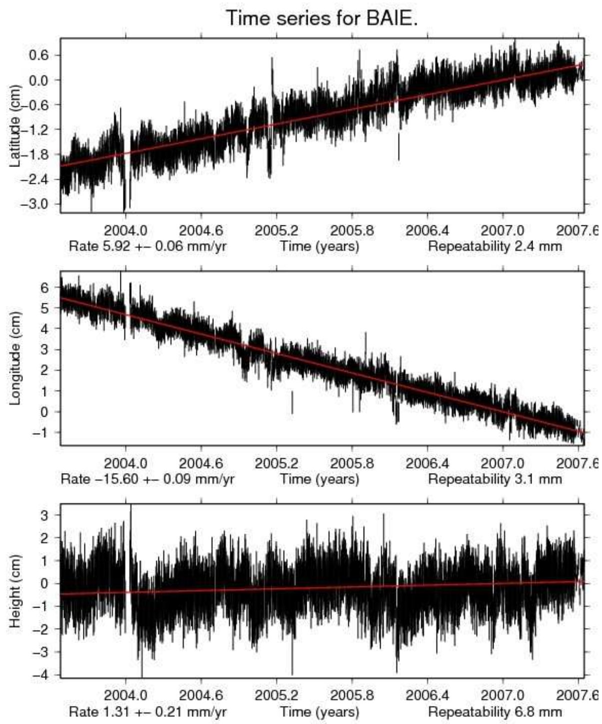


Figure 7: Baie-Comeau

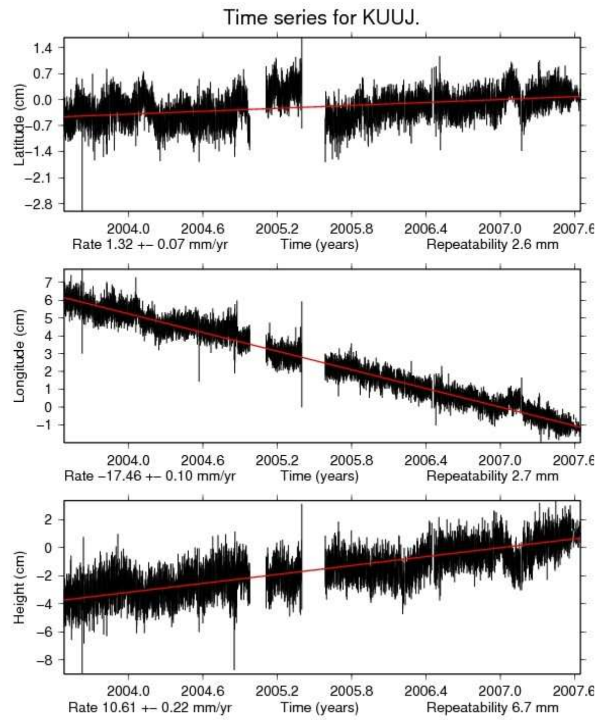


Figure 8: Kuujjuarapik

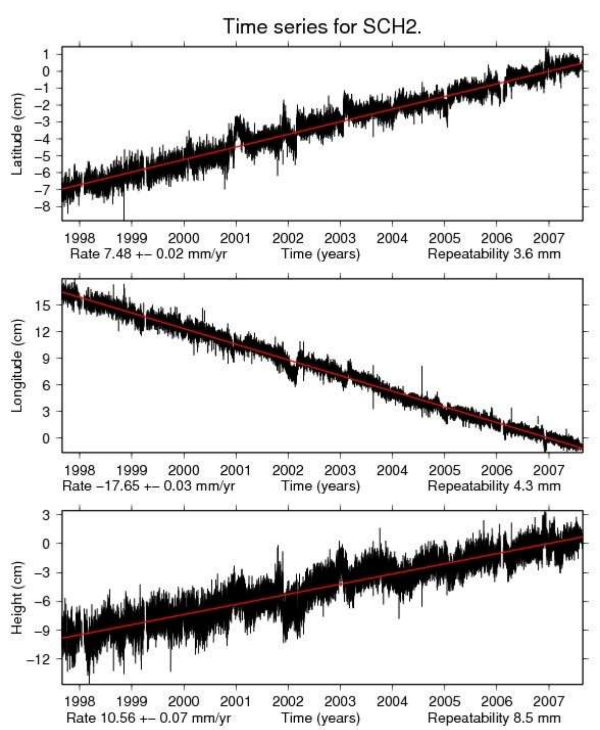


Figure 9: Schefferville

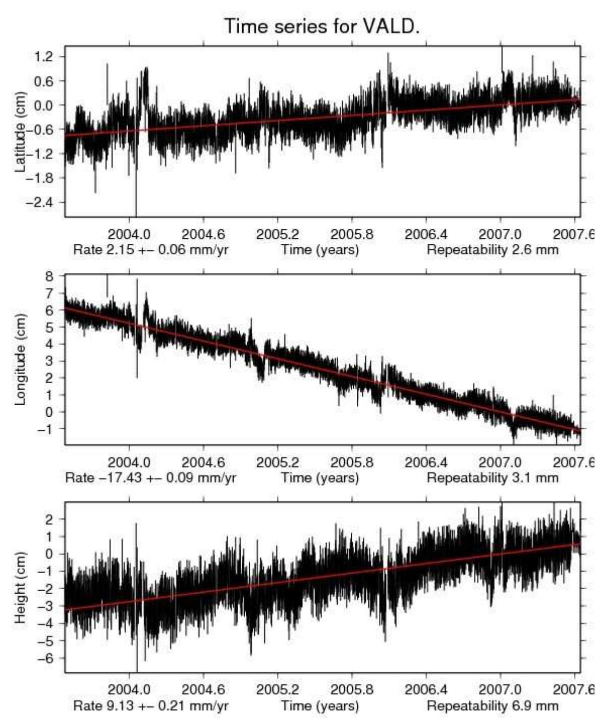


Figure 10: Val-d'Or

3 Coordinate Systems

To locate an object mathematically in a terrestrial reference system, it is necessary to use a coordinate system. In geodesy, there are mainly two three-dimensional coordinate systems: Cartesian and curvilinear. In the Cartesian coordinate system, the position of a point is defined by three orthogonal Cartesian coordinates (X, Y, Z). Geocentric by convention, the Z axis coincides with the Earth's axis of rotation, the XY plane coincides with the plane of the equator, the X axis points to the prime meridian and the Y axis is perpendicular to the XZ plane (Figure 11).

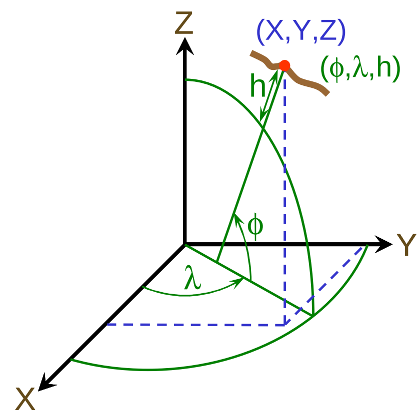


Figure 11: Cartesian and geodetic coordinates

In the curvilinear coordinate system, the position of a point is defined with respect to a curved reference surface using three curvilinear coordinates. In geodesy, this surface is an ellipsoid of revolution. This mathematical surface most resembles the shape of the geoid (Figure 12). The geoid is described in Chapter 6. The size of the ellipsoid is defined by the semi-major axis (a) and its shape by flattening (f). The semi-minor axis (b) is also used to define the shape of the ellipsoid. It is the result of the following relation: b = a (1 – f).

In geodesy, the curvilinear coordinates are called "geodetic coordinates," and are latitude (ϕ), longitude (λ) and height (h) (Figure 11). Latitude and longitude are sometimes called "geographic coordinates." The curvilinear coordinate system makes it possible to differentiate the horizontal component (ϕ, λ) from the height component (h). The Cartesian coordinate system does not offer this possibility.

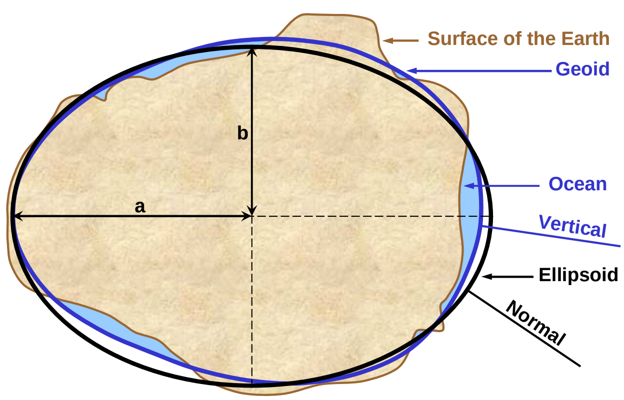


Figure 12: Ellipsoid and geoid

The Cartesian and geodetic coordinate systems are interrelated by the following relation:

$$(X, Y, Z) \leftarrow \text{Transformation} \rightarrow (\phi, \lambda, h)$$

Most geodetic references describe the transformation from one coordinate system to another in detail.

The height from a geodetic coordinate system is called "geodetic height." It represents the distance between the surface of the ellipsoid and an object along the normal. The expression "ellipsoidal height" is sometimes found in the literature to designate geodetic height.

4 Conventional Geodetic Datums

Most conventional reference systems were established between the late 19th century and the middle of the 20th century for national requirements, and some for continental requirements. They were realized based primarily on horizontal angles and a few distances since electronic distance measurements (EDM) appeared in the 1950s. The large majority of distance measurements used in existing networks was used to densify and not realize conventional reference systems. These conventional reference systems are commonly called “geodetic datum” or “horizontal datum.”

Deployment of a conventional reference system is generally based on the following two steps: definition (the system) and realization (the frame).

Defining a system consists in establishing the set of parameters and conventions that will be used to determine a position in this datum. The parameters and conventions include the choice of ellipsoid and its fundamental point, also called the “initial point.” The shape and dimension of the ellipsoid are selected to coincide very closely with mean sea level of the area covered by the conventional reference system. Depending on the area covered, the ellipsoid’s shape and dimension vary, as well as its origin in relation to the centre of the Earth.

Generally located in the centre of the network, the fundamental point is used to define the position and orientation of the ellipsoid in space with the following parameters:

- ϕ_0 : geodetic latitude
- λ_0 : geodetic longitude
- N_0 : geoid undulation
- ξ_0 : north-south component of deflection from the vertical
- η_0 : east-west component of deflection from the vertical
- $\delta\alpha_{0j}$: Laplace’s equation

Geoid undulation represents the geodetic height of the geoid at one point. The parallelism between the ellipsoid’s axis of rotation and the Earth’s is provided by Laplace’s equation (Vaníček and Krakiwsky, 1982):

$$\delta\alpha_{0j} = A_{0j} - \alpha_{0j} = \eta_0 \tan(\phi_0) + [\xi_0 \sin(A_{0j}) - \eta_0 \cos(A_{0j})] \operatorname{cotan}(z_{0j}) \quad (1)$$

Where:

A_{0j} : astronomic azimuth to a point j

α_{0j} : geodetic azimuth to a point j

z_{0j} : zenith angle to a point j

By convention, the ellipsoid is tangent to the geoid at the fundamental point. Thus, the geoid undulation is null and the normal and vertical (Figure 12) are parallel. Therefore:

$$N_0 = 0$$
$$\xi_0 = 0$$
$$\eta_0 = 0$$

Considering these conditions, the astronomic (Φ_0, Λ_0) and geodetic (ϕ_0, λ_0) coordinates, and the astronomic (A_{0j}) and geodetic (α_{0j}) azimuths to point j are identical since:

$$\xi_0 = \Phi_0 - \phi_0 = 0$$
$$\eta_0 = (\Lambda_0 - \lambda_0)\cos(\phi_0) = 0$$

Realization consists in giving concrete expression to the datum by defining the points that will form the geodetic network, taking measurements (angles, distances, astronomic observations) and determining the coordinates (adjusting observations) of the points. When defining the points, the values at the fundamental point can be modified if astronomic observations were taken at several points evenly distributed throughout the network.

NAD27

The conventional reference system that was implemented in North America is NAD27 (North American Datum of 1927). It was used as a basis for all work requiring a location from the early 1930s until the early 1990s. NAD27 used the Clarke ellipsoid of 1866 and its fundamental point was Meades Ranch (Kansas) located approximately in the centre of the US. The Clarke ellipsoid of 1866 was to coincide with the geoid for the North American continent and be tangent to it at the fundamental point. Data processing ended in 1932. Work to densify the network started from that point forward and lasted for more than 50 years.

Densification introduced random and systematic errors as the work progressed. The main factors responsible for these errors were the shortcomings in the geometry of the network and the absence of a precise geoid model. A lack of consistency in the adjustment methods also contributed to propagate the errors in the network (Pinch, 1990).

While NAD27 was being established and densified until the 1950s, the main instrument for measuring distance was the invar tape. The quality, and especially the scarcity, of measured distances before the era of EDM introduced distortions in the scale of the network. Doubt was also cast on the shape of the triangulation networks. A combination of all these factors created a relative error of about 27 m on either side of Hudson's Bay. Compared to the fundamental point, the relative error could reach 100 m in the far northeast and northwest areas of Canada.

The absence of accurate geoid models in the calculations also introduced distortions in the scale of the network. To calculate coordinates, all distances must be reduced on the ellipsoid. Reducing distances requires the geodetic height (h) to be known at each point. Since, by definition, the ellipsoid coincided with the geoid, the geoid undulation (N) was almost always ignored, therefore $h \approx H$. This approximation introduced an error of scale of 1 ppm in the distance for each discrepancy of 6.4 m of undulation ignored in reducing the distances.

In the 1970s, in order to decrease the distortions in the NAD27 frame in its area, the Service de la géodésie (geodetic service) of the Quebec Department of Lands and Forests had the entire Quebec geodetic network adjusted by keeping the points in the province's southwest fixed. This adjustment, called the "Compensation géodésique du Québec de 1977" (CGQ77) (Quebec geodetic adjustment of 1977), was the second realization of NAD27 in Quebec. To distinguish it from the first, the geodetic service called it NAD27 (CGQ77). This new realization created discrepancies as large as 15 m between NAD27 and NAD27 (CGQ77) in the eastern part of the province.

5 Modern Geodetic Datums

A terrestrial reference system is a mathematical concept, and therefore theoretical. It is a set of parameters and conventions making it possible to describe the position of a point in space. The concrete realization of this concept is called a “terrestrial reference frame.” It provides a physical representation of the terrestrial reference system through a set of physical points related to the Earth’s crust. The position of each point is known with a high degree of accuracy in the terrestrial reference system.

The ideal terrestrial reference system would have the following characteristics:

- It is an axis system that rotates with the Earth;
- Its origin (O) coincides with the centre of mass of the Earth;
- The axis of the system are orthogonal (X, Y, Z);
- The scale of measurement is the metre.

There are two categories of terrestrial reference systems: global and regional.

5.1 Global Terrestrial Reference Systems

The global terrestrial reference system is unique for the Earth. Its main characteristic is its independence in relation to the tectonic plates. When the global terrestrial reference frame is realized, the position of the plates and its fundamental points (▲) is frozen at a specific epoch called the “reference epoch” (t_0). Figure 13 shows the position of the North American continent at epoch t_0 = 1997.0, or January 1, 1997. At observation epoch (t), the plates and their markers have moved, but the terrestrial reference frame remains in the same place as at determination epoch t_0. Figure 14 shows the position of the North American continent at observation epoch t = 2008.4. For the fundamental points, their position at epoch t is calculated based on their initial position at reference epoch (t_0) and their velocity using the following equation:

$$\vec{\mathbf{X}}(t) = \vec{\mathbf{X}}(t_0) + \dot{\vec{\mathbf{X}}}(t - t_0) \qquad (2)$$

where $\vec{\mathbf{X}}$ is the position vector and $\dot{\vec{\mathbf{X}}}$ is the velocity vector. The velocity vector is estimated at the same time as the position of the points when the terrestrial reference frame is realized.

The position of the new points (▲) is calculated from the position of the fundamental points at epoch *t*. If new observations are carried out at epoch 2009.3, it is then possible to determine the displacement of the new points.

For some requirements, it is necessary to return to the reference epoch (t_0) (Figure 15). The position of the fundamental points in the terrestrial reference frame is already known at epoch t_0. There is no velocity vector for the points (▲) established at epoch *t*. Their position at epoch t_0 is obtained by applying the velocity from a plate motion model. Such models are generally produced when the terrestrial reference frame is realized. Independent models also exist, such as NNR-NUVEL-1A (DeMets et al., 1994). NUVEL-1A (Northwestern University Velocity, model 1, modification A) is a relative motion model for lithospheric plates on the surface of the globe. The NNR (No-net-rotation) condition makes it possible to express the motion in absolute terms where the mean motion is null. Use of a plate motion model such as NNR-NUVEL-1A is limited to horizontal motion. Local and vertical motion, such as post-glacial rebound or subsidence, is not taken into account in this type of model.

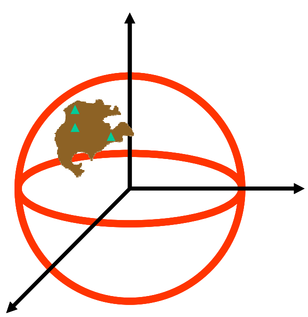


Figure 13:
Origin epoch 1997.0

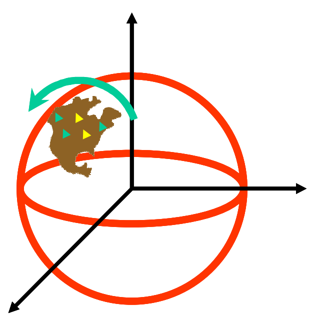


Figure 14:
Observation epoch 2008.4

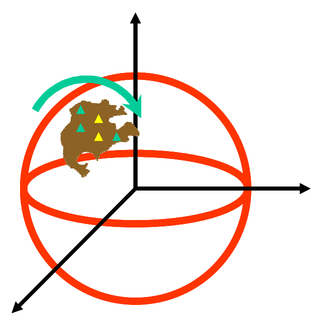


Figure 15:
Return to epoch 1997.0

It is also possible to transform the positions from one terrestrial reference frame to another with a different determination epoch. Going back and forth between frames and epochs is becoming increasingly complex and requires precise knowledge of the motion (velocity). Time therefore becomes the fourth component in a global terrestrial reference system.

The link between two terrestrial reference frames is obtained by a 7+7 parameter transformation, or three translations (T_X , T_Y , T_Z), scaling (S) and three rotations (ε_X, ε_Y, ε_Z), as well as their respective time derivatives ($\dot{T}_X$, $\dot{T}_Y$, $\dot{T}_Z$, $\dot{S}$, $\dot{\varepsilon}_X$, $\dot{\varepsilon}_Y$, $\dot{\varepsilon}_Z$).

The general expression for transforming the position of reference system A to reference system B is:

$$\vec{\mathbf{X}}_B = \vec{\mathbf{X}}_A + \vec{\mathbf{T}} + S\vec{\mathbf{X}}_A + \mathbf{R}\vec{\mathbf{X}}_A \tag{3}$$

$$\vec{\mathbf{X}} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \quad \vec{\mathbf{T}} = \begin{pmatrix} T_X \\ T_Y \\ T_Z \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 0 & -\varepsilon_Z & \varepsilon_Y \\ \varepsilon_Z & 0 & -\varepsilon_X \\ -\varepsilon_Y & \varepsilon_X & 0 \end{pmatrix}$$

Where:

$\vec{\mathbf{X}}_A$	:	Position vector in frame A
$\vec{\mathbf{X}}_B$	:	Position vector in frame B
$\vec{\mathbf{T}}$	:	Translation vector
S	:	Scale factor
$\mathbf{R}$	:	Rotation matrix

The time component is applied to each parameter (P) as follows:

$$\mathrm{P}(t) = \mathrm{P}(t_0) + \dot{\mathrm{P}}(t - t_0) \tag{4}$$

Where $\dot{\mathrm{P}}$ is the temporal variation of the parameter between epochs t_0 and t. Equations (3) and (4) are used for all types of terrestrial reference systems.

The best known global terrestrial reference systems are the International Terrestrial Reference System (ITRS) and the World Geodetic System of 1984 (WGS84).

5.1.1 ITRS

The International Earth Rotation and Reference Systems Service (IERS) (http://www.iers.org/) was established in 1987 by the International Astronomical Union (IAU) and the International Union of Geodesy and Geophysics (IUGG) to replace the International Time Bureau (BIH) on January 1, 1988. Its primary objective is to serve the astronomy, geodetics and geophysics communities by providing them with terrestrial and celestial reference systems, among other things, and realizing these systems. The International Terrestrial Reference System is a result of this mandate.

The main characteristics of the ITRS (IERS, 2004) are:

- The origin coincides with the centre of mass of the Earth, including the oceans and atmosphere;
- The unit of length is the metre;

- The orientation of the axes is the one established by the BIH (epoch 1984.0).

The International Terrestrial Reference Frame (ITRF) is the concrete realization of the ITRS (http://itrf.ensg.ign.fr/). This frame is established by a global network of several hundred sites that have one or more tracking stations.

Since 1988, the IERS has co-ordinated several realizations of the ITRF called ITRF*y* (t_0), where *y* represents the year of the most recent observations used for the calculations, and t_0 is the reference epoch. For each realization, the IERS publishes a technical note describing its implementation. In addition, for the tracking network stations, it also gives their position (X, Y, Z and standard deviations σ_X, σ_Y, σ_Z in metres) and their velocity ($\dot{X}$, $\dot{Y}$, $\dot{Z}$ and standard deviations $\sigma_{\dot{X}}$, $\sigma_{\dot{Y}}$, $\sigma_{\dot{Z}}$ in metres per year) at the reference epoch. The velocity contains, among other things, the drift velocity of the tectonic plates and the post-glacial rebound for the northern regions. These phenomena are the most important with respect to the absolute displacement of a station. The reference ellipsoid used to express the position in geodetic coordinates is GRS80 (Geodetic Reference System of 1980) (Moritz, 1988).

The measurements to realize the ITRF come from the following space-based positioning techniques:

- VLBI: Very Long Baseline Interferometry;
- SLR: Satellite Laser Ranging;
- DORIS: Doppler Orbitography and Radiopositioning Integrated by Satellite;
- GPS: Global Positioning System;
- LLR: Lunar Laser Ranging.

The LLR technique contributed to ITRF solutions until 2000. By combining these techniques, the strengths and weaknesses of each contribute to implementing a frame that complies with the ITRS characteristics.

In April 2010, the most recent realization is ITRF2005 (2000.0), or the 11th since deployment of the IERS in 1988. It has been in effect since October 2006. This frame is established by a global network of some 500 stations distributed among 300 or so sites (Figure 16) (ITRF, 2008a). Sites can accommodate up to four space-based techniques.

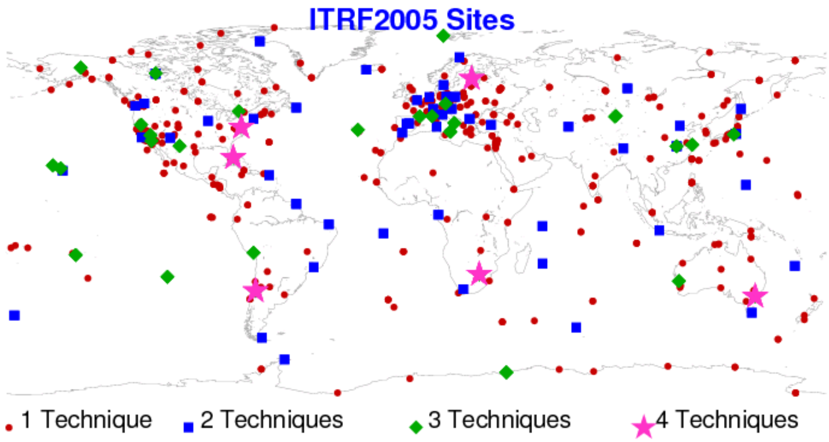


Figure 16: Network of ITRF stations

The data that were used to realize ITRF2005 come from time series of station positions and Earth orientation parameters calculated by different international services associated with a space-based technique:

- Weekly solution
    - International GNSS Service (IGS) (GNSS: Global Navigation Satellite System) (http://igscb.jpl.nasa.gov/);
    - International Laser Ranging Service (ILRS) (http://ilrs.gsfc.nasa.gov/);
    - International DORIS Service (IDS) (http://ids-doris.org).
- Daily solution
    - International VLBI Service (IVS) (http://ivscc.gsfc.nasa.gov/).

Taking the strength of each technique into account, ITRF2005 was defined as follows:

- The scale and its temporal variation are aligned such that they are null compared with the IVS time series at epoch 2000.0;
- The translations and their temporal variation are aligned such that they are null compared with the ILRS time series at epoch 2000.0;
- The orientation and its temporal variation are aligned such that they are null compared with ITRF2000 at epoch 2000.0.

Appendix A presents the summary of an article describing ITRF2005. The IERS is working on the next version of the ITRF. Deployment of ITRF2008 (2005.0) is planned for summer 2010.

5.1.2 WGS84

The World Geodetic System WGS84 was developed and implemented by the Defence Mapping Agency (DMA) in the early 1980s specifically for the GPS. Today, the National Geospatial-Intelligence Agency (NGA) (www.nga.mil) in the US is the organization that coordinates the maintenance of WGS84.

The main characteristics of WGS84 (NIMA, 1997) are:

- The origin coincides with the centre of mass of the Earth, including the oceans and atmosphere;
- The unit of length is the metre;
- The orientation of the axes is the one established by the BIH (epoch 1984.0).

Since its implementation in the 1980s, several realizations of WGS84 have been coordinated by the NGA and its predecessors. Contrary to the ITRS, the frame that establishes WGS84 has the same name.

The first realization of WGS84 comes from a set of points established by TRANSIT in the 9Z-2 reference system from the Naval Surface Warfare Center (NSWC), and then aligned to BTS84. A predecessor of the GPS, TRANSIT used Doppler measurements to obtain a position. Associated with this type of measurement, it was called the “Doppler system.” Global terrestrial reference system BTS84 was realized by the BIH in 1984 from measurements coming from the VLBI, SLR, LLR and TRANSIT space-based techniques. This first realization of WGS84 is sometimes called by the name of the initial version, or the Doppler version.

The WGS84 ellipsoid was specially developed for WGS84. Among other things, it is used to express positions in geodetic coordinates. Thus, WGS84 refers to the system, the frame and the ellipsoid. This triple definition causes some confusion among users. Furthermore, the WGS84 ellipsoid is almost identical in size to the GRS80 ellipsoid (Table 2). This is another source of confusion among users.

Table 2: WGS84 and GRS80 ellipsoids

Parameter	WGS84	GRS80
Semi-major axis (a)	6,378,137.0000 m	6,378,137.0000 m
Semi-minor axis (b)	6,356,752.3142 m	6,356,752.3141 m
Flattening (f)	1/298.257223563	1/298.257222101

Improvements to international GPS networks and terrestrial reference systems showed that the Doppler version of WGS84 was almost geocentric. Table 3 (ITRF, 2008b) presents the transformation parameters between ITRF90 and the Doppler version of WGS84. The discrepancy between the origins of ITRF90 and ITRF2000 is about 5 cm. Therefore, the discrepancy between the origin of the Doppler version of WGS84 and the centre of mass of the Earth would be about 0.5 m.

Table 3: From ITRF90 to WGS84 (Doppler)

Parameter	Value
Translation X (m)	0.060
Translation Y (m)	-0.517
Translation Z (m)	-0.223
Rotation X (″)	0.0183
Rotation Y (″)	-0.0003
Rotation Z (″)	0.0070
Scale (ppm)	-0.011

To ensure sufficient accuracy of WGS84, new realizations were subsequently carried out (Table 4). To differentiate them, the GPS week from which the NGA started calculation of the precise ephemeris is added to the name. The dates in Table 4 indicate when the realization was implemented in the orbits broadcast by the GPS satellites. Before January 23, 1987, the broadcast orbits were referenced in WGS72. Table 4 also shows which terrestrial reference frame the realization is equivalent to and its degree of accuracy in relation to this frame. Today, the WGS84 frame is established by more than a dozen stations scattered around the planet. Considering its degree of accuracy in relation to the ITRF2000 solution, WGS84 is actually equivalent to ITRF2000.

Table 4: Evolution of the WGS84 frame

Name	From	To	Equivalence	Accuracy
WGS84	1987/01/23	1994/06/28	BTS84	±100 cm
WGS84 (G730)	1994/06/29	1997/01/28	ITRF92 (1994.0)	±20 cm
WGS84 (G873)	1997/01/29	2002/01/19	ITRF94 (1997.0)	±10 cm
WGS84 (G1150)	2002/01/20	–	ITRF2000 (2001.0)	±2 cm

5.2 Regional Terrestrial Reference Systems

In regional terrestrial reference systems, the coordinate system is fixed to a tectonic plate. For instance, NAD83 (North American Datum of 1983) is fixed to the North American plate and ETRS89 (European Terrestrial Reference System 1989) is fixed to the Eurasian plate. Since the system is fixed to the tectonic plate, it rotates with the plate. Thus, the coordinates do not vary in time. Only local phenomena can change the position of a point.

Figures 17 and 18 show the rotation of a regional terrestrial reference system compared with a global system. Figure 17 shows the situation of the North American plate compared with the coordinate system on December 31, 1983. The regional and global systems coincide. Twenty-four years later, on December 31, 2007, the plate is no longer in the same place (Figure 18), but the coordinates have not changed because the regional terrestrial system and the plate are one and the same. However, in the global system, the coordinates are no longer the same. The difference in coordinates between these two epochs would represent a displacement of about 48 cm, since the velocity of the plate in Canada is about 2 cm/yr.

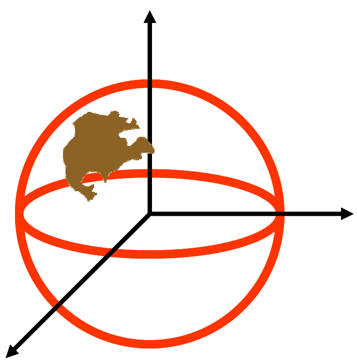


Figure 17: North American plate at 1983-12-31

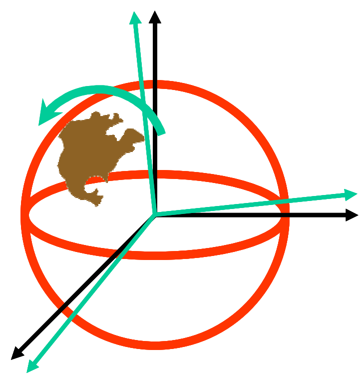


Figure 18: North American plate at 2007-12-31

5.2.1 NAD83

The terrestrial reference system in use in North America is NAD83. It replaced the NAD27 conventional reference system in the early 1990s for the reasons that follow. The emergence of space-based positioning techniques in the 1960s and 1970s and the use of increasingly accurate EDM showed that NAD27 no longer met the accuracy and criteria required for these techniques, especially space-based positioning. For instance, its origin was about 250 m from the centre of mass of the Earth. It was time for NAD27 to make way for a modern datum.

The main characteristics of NAD83 are:

- The origin coincides with the centre of mass of the Earth, including the oceans and atmosphere;
- The unit of length is the metre;
- The orientation of the axes is the one established by the BIH (epoch 1984.0).

As mentioned above, NAD83 is fixed to the stable part of the North American plate at a determined epoch (December 31, 1983).

There are two major realizations of this system: NAD83 (1986) and NAD83 (CSRS). The term in parentheses is used to differentiate the two realizations. The absence of this term designates the system.

5.2.1.1 NAD83 (1986)

NAD83 (1986), also called “original NAD83,” is a conventional (planimetric) reference frame adjusted in 1986 (Schwarz, 1989). This frame is the result of a multi-source North American adjustment of more than 266,400 points including some 7,500 points in the primary Canadian network. The data came from three techniques: conventional observations (angles, distances, astronomical azimuths), positions obtained with the TRANSIT positioning system (commonly called “Doppler point”) and the VLBI technique. The contribution from each of these techniques in the adjustment is:

- Close to 1,780,000 conventional observations;
- More than 1,300 TRANSIT positions;
- Close to 50 VLBI positions.

To adjust this mass of data, the Helmert block method was chosen. It was necessary to form close to 340 Helmert blocks in order to estimate more than 928,700 parameters. The calculations were made on the GRS80 ellipsoid.

Transformations were applied to the three techniques to make them compatible with each other and to align them to BTS84. The transformation parameters applied to the different techniques are listed in Table 5 and the rotations are shown in Figure 19 (Schwarz, 1989). The TRANSIT positions underwent the same transformation as those used to make WGS84 geocentric. As with the ITRF, the VLBI provided the scaling.

Table 5: Transformation parameters

Parameter	Conventional	TRANSIT	VLBI
Translation X (m)	–	0	0
Translation Y (m)	–	0	0
Translation Z (m)	–	4.5	0
Rotation X (″)	–	0	0.020
Rotation Y (″)	–	0	0.020
Rotation Z (″)	-0.365	-0.814	0.010
Scale (ppm)	-0.237	-0.6	-0.075

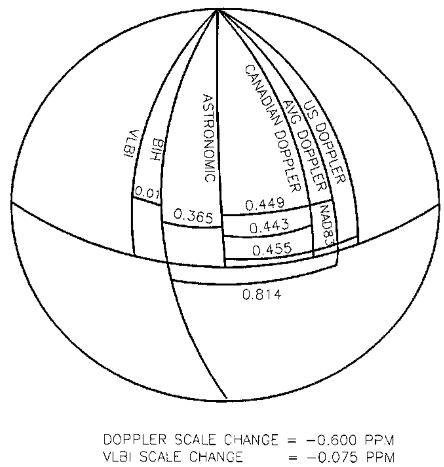


Figure 19: Rotation of the systems

After the North American adjustment was completed, Canada adjusted close to 105,000 points from the primary and secondary provincial networks (Pinch, 1990). Quebec then adjusted its 3rd and 4th order networks in the early 1990s.

NAD83 made it possible to eliminate dozens of metres of relative or absolute positioning errors in NAD27. The Geodetic Survey Division (GSD) of Natural Resources Canada estimates that the absolute accuracy of NAD83 (1986) varies from a few centimetres to over 2 m, with an average of 0.3 m (GSD, 2008a). The relative accuracy is better than NAD27 or NAD27 (CGQ77) in Quebec.

In NAD83 (1986), the hierarchy in the networks stayed the same as the one used in NAD27. Table 6 shows the hierarchy for the conventional NAD27 and NAD83 (1986) networks as they were established in Quebec.

Table 6: Conventional infrastructure in Quebec

	Order										
Network	1			2			3			4	
Environment	A	B	C	A	B	C	A	B	C	A	B
Average distance between points (km)	8	20	30	2.5	6	15	0.75	1.75	2.5	0.25	0.5
Minimum distance between points (km)	5	12	20	1.5	4	10	0.4	1	1.5	0.1	0.2

Network
1: Primary
2: Secondary
3: Tertiary
4: Complementary

Environment
A: Urban
B: Semi-urban
C: Not developed

A metric quality criterion is associated with the position of each geodetic point. The Direction de la référence géodésique (DRG) at the Quebec Department of Natural Resources and Wildlife uses four categories, or classes, to describe the metric quality of the coordinate. Coordinates that do not meet the metric criteria are not classified (class D) and are not available for users. The classification is done based on the standard deviation (σ) of the position resulting from the data adjustment. Table 7 shows the maximum permitted limit in latitude and longitude for a given class and order. Orders IA, IB and IC are used for the points established by intersection.

Table 7: Classification criteria (m) in the conventional infrastructure

		Order													
		1A	1B	1C	2A	2B	2C	3A	3B	3C	4A	4B	IA	IB	IC
Class	1	0.060	0.120	0.250	0.040	0.075	0.200	0.030	0.050	0.080	0.020	0.035	0.050	0.100	0.300
	2	0.085	0.170	0.354	0.057	0.106	0.283	0.042	0.071	0.113	0.028	0.049	0.071	0.141	0.424
	3	0.120	0.240	0.500	0.080	0.150	0.400	0.060	0.100	0.160	0.040	0.070	0.100	0.200	0.600

NAD83 (1986) and GPS

The implementation of NAD83 (1986) was completed as GPS quickly gained ground. Starting in the late 1980s, the internal weaknesses of NAD83 (1986) were beginning to appear with the increasing use of GPS. One of the first observations was this frame's lack of compatibility with the space-based techniques. Despite the adjustments to conventional observations in a modern geocentric frame, NAD83 (1986) remains above all a "planimetric datum." But the GPS, or any other space-based technique, operates in a three-dimensional environment. The absence of geodetic height (h) creates a problem for integrating GPS measurements into NAD83 (1986). It is becoming clear that it is necessary to rely on a geoid model to obtain a geodetic height at points that have a known orthometric height (H). Chapter 6 on vertical datums presents the limitations of using an orthometric height and a geoid model to obtain a geodetic height.

Another observation was that NAD83 (1986) was not as geocentric as it should be. The origin is about 2.2 m from the centre of mass of the Earth. This offset is caused primarily by the inaccuracy of the Doppler points that were used as control points when the conventional observations were adjusted. The positions established by the TRANSIT system had an accuracy of about 1 m (3D). Since the control points were located only on the continent, this also contributed to offsetting the origin.

The accuracy of GPS in relative positioning is far superior to conventional observations for geodetic requirements. GPS highlighted the heterogenous quality of conventional observations caused by the inaccuracy of conventional instruments. Recall that the conventional observations that were used in the adjustment were made over a long period of time with several generations of instruments. GPS also brought out the weaknesses in the frame of the conventional networks given the absence of measurements between points or polygons.

These observations, combined with the emergence of global terrestrial reference systems, pushed the North American geodetic organizations to review the NAD83 frame implemented several years earlier.

5.2.1.2 NAD83 (CSRS)

In the early 1990s, because of the weaknesses in the NAD83 (1986) frame, a decision needed to be made to give North America a modern frame that was compatible with space-based technologies and the different ITRF realizations. Canada and the US agreed to realize this new frame. In Canada, this realization is called NAD83 (CSRS), where CSRS stands for “Canadian Spatial Reference System.” In the US, this frame is called NAD83 (NSRS), where NSRS stands for “National Spatial Reference System.”

To implement this new frame, two solutions were available to these two countries. One solution was to abandon the newly established reference system and replace it with the ITRS. This was the approach chosen by Europe to replace its conventional geodetic datum ED50 (European Datum of 1950) and its ED79 and ED87 versions with a regional terrestrial reference system, in this case the European Terrestrial Reference System of 1989 (ETRS89). Europe fixed ITRF89 (1989.0) to the stable part of the Eurasian plate to create ETRF89 (1989.0).

The other solution was to tie NAD83 to the ITRS as closely as possible based on common VLBI stations. These stations would be used to estimate transformation parameters between two separate realizations. See section 5.2.1.3.

While it was considering the improvements to NAD83, GSD decided to maintain and even improve its network of tracking stations that continuously track GPS satellites. This network would form the basis for a new infrastructure with an active component made up of the tracking stations, and a passive component to be implemented. The active component is called the Canadian Active Control System (CACS). Today, approximately 50 tracking stations make up the CACS (Figure 20) (GSD, 2008b). Some of them are part of the ITRF’s network of tracking stations. They thus have positions in various ITRF solutions. With the transformation parameters that link NAD83 and the ITRS, it is therefore possible to give a NAD83 (CSRS) position to these tracking stations.

Figure 20: Canadian Active Control System

Figure 21: Canadian Base Network

The passive component, which is the result of collaboration with the provinces, is a network consisting of about 160 points established by concrete pillars mounted with a forced centring plate (Figure 21) (GSD, 2008c). Called the Canadian Base Network (CBN), the position of the points in this network is determined with great accuracy in the ITRS thanks to the GPS measurements taken simultaneously with the CACS. With the transformation parameters that link NAD83 and the ITRS, it is thus possible to give a NAD83 (CSRS) position to these points.

GSD estimates that the absolute accuracy of NAD83 (CSRS) is of the order of a few centimetres or better, corresponding to the accuracy of the recent versions of the ITRF (GSD, 2008a). Figure 22 (Craymer, 2006a) shows the internal errors of NAD83 (1986) obtained by the difference between the NAD83 (CSRS) position and the NAD83 (1986) position across Canada. Globally, these errors seem random. However, locally, they can be systematic. It is important to remember that NAD83 (1986) and NAD83 (CSRS) are two realizations that are fundamentally different from the same NAD83 terrestrial reference system.

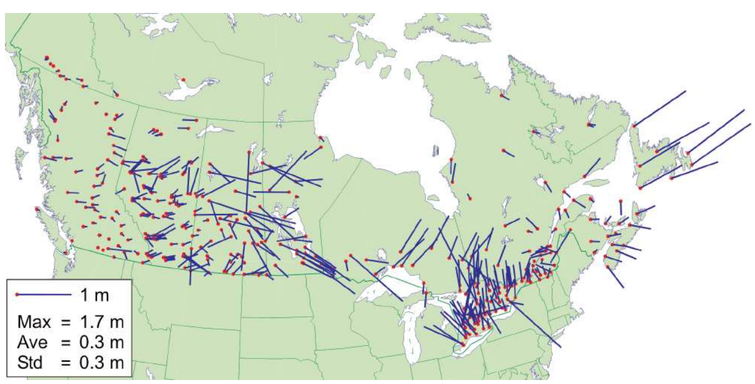


Figure 22: Position errors in NAD83 (1986)

To distinguish the conventional NAD83 (1986) infrastructure from the NAD83 (CSRS) spatial infrastructure in Quebec, the DRG implemented a new hierarchy (Table 8).

Table 8: NAD83 (CSRS) infrastructure in Quebec

Level	Equivalence	Distance Between Points	Method
High Precision Network			
AA	CACS	1,000 to 1,500 km	GPS
A1	CBN	100 to 500 km	GPS
A2		about 50 km	GPS
A3		5 to 15 km	GPS
Densification Network			
B1		more than 5 km	GPS
B2		less than 5 km	GPS
Conventional Network			
C1	Primary network	20 km	Conventional
C2	Secondary networks	5 km	Conventional
C3	Tertiary networks	1 to 2 km	Conventional
C4	Urban networks	200 to 500 m	Conventional
CI	Intersected points		Conventional

The concept of “order” was replaced by “level.” The A level constitutes a high precision network. More than 1,900 points form this network. The A2 level is a first densification of the CBN that is called A1 in the Quebec hierarchy. Over time, GSD has converted several CBN points into CACS stations. In the Quebec infrastructure, these points remain in their original A1 level. The B level is the densification network and consists of more than 20,000 points. The operating procedure for establishing the B level is less rigorous than the procedure for the A2 and A3 levels, but the metric quality is just as good.

The C level, which is the conventional network, is the non-spatial part of the NAD83 (CSRS) infrastructure in Quebec. The positions of some 41,000 points at this level come from the adjustment of conventional observations based on points from the A and B levels. This adjustment produced a purely planimetric frame. The C level points therefore have no geodetic height in NAD83 (CSRS). Consequently, for work using GPS technology, only the A and B level points are recommended. The CI level contains points determined by the intersection method. Church bell towers represent most of the points at the CI level.

Table 9 gives the classification criteria used by the DRG for its entire NAD83 (CSRS) infrastructure. The criteria are based on the standard deviation (σ) resulting from the geodetic adjustment. The planimetric classification criterion shows that the latitude (ϕ) <u>and</u> longitude (λ) meet this criterion. For instance, if the latitude is class 1 and the longitude is class 2, then the point will be class 2 in planimetry. Coordinates that do not meet these metric criteria are not classified (class D) and cannot be accessed by users. Of the 41,000 points adjusted in the C level, close to 31,800 meet the criterion for classes 1, 2 or 3.

Table 9: NAD83 (CSRS) classification criteria

	Class			
	1	**2**	**3**	**D**
Planimetry (ϕ, λ)	σ ≤ 0.01 m	0.01 m < σ ≤ 0.03 m	0.03 m < σ ≤ 0.06 m	0.06 m < σ
Height (h)	σ ≤ 0.02 m	0.02 m < σ ≤ 0.06 m	0.06 m < σ ≤ 0.10 m	0.10 m < σ

5.2.1.3 ITRF — NAD83 (CSRS) Link

During the adjustment that determined the positions in NAD83 (1986), only the stations resulting from the VLBI and TRANSIT space-based techniques obtained three-dimensional positions. Of these two techniques, VLBI is the only one used in realizing the ITRF. Thus, some VLBI stations have a three-dimensional position in NAD83 (1986) and in ITRF realizations. It is therefore possible to estimate the transformation parameters at a given epoch. An ITRF position can thus be transformed to a NAD83 (CSRS) position.

The first transformation is based on the ITRF89 and NAD83 (1986) positions of the 11 VLBI stations located in the US (Soler et al., 1992). It confirmed the discrepancy of about 2.2 m between the origin points of these two frames.

The second transformation based on ITRF96 (1997.0) is the result of a Canada-US collaboration (Craymer et al., 2000). The seven parameters (three translations, three rotations and one scale factor) of the transformation (Table 10) were determined from 12 North American VLBI stations (Figure 23) (Soler and Snay, 2004). Thereafter, these parameters became the official link with the ITRF for North America at epoch 1997.0. However, since the ITRF is a datum that is independent from the Earth's crust, the motion of the North American tectonic plate must be taken into account if the epoch of the work is different from 1997.0 in ITRF96. The NNR-NUVEL-1A plate motion model is used to determine the displacement of the North American plate between two epochs. This time element is introduced into the transformation by adding three additional rotation parameters (Table 11).

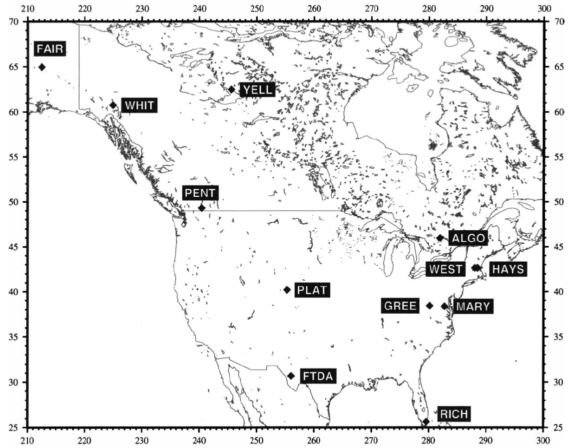


Figure 23: VLBI stations in NAD83

Table 10: ITRF96 (1997.0) → NAD83 (CSRS) transformation parameters

Parameter	Value
T_X	0.9910 m
T_Y	-1.9072 m
T_Z	-0.5129 m
ε_X	-25.790 m″
ε_Y	-9.650 m″
ε_Z	-11.660 m″
S	0.00 ppb
m″: milliarcsecond ppb: parts per billion (10^{-3} ppm)	

Table 11: NNR-NUVEL-1A rotation parameters

Parameter	Value (milliarcsecond/year)
$\dot{\varepsilon}_X$	-0.0532
$\dot{\varepsilon}_Y$	0.7423
$\dot{\varepsilon}_Z$	0.0316

The mathematical expression for the ITRF96 to NAD83 (CSRS) transformation is:

$$\vec{\mathbf{X}}^{N} = \vec{\mathbf{T}} + \left[\mathbf{R} + \dot{\mathbf{R}}^{N}(t - t_0)\right]\vec{\mathbf{X}}^{I}_{96}(t) \quad (5)$$

$$\dot{\mathbf{R}}^{N} = \begin{pmatrix} 0 & -\dot{\varepsilon}_Z & \dot{\varepsilon}_Y \\ \dot{\varepsilon}_Z & 0 & -\dot{\varepsilon}_X \\ -\dot{\varepsilon}_Y & \dot{\varepsilon}_X & 0 \end{pmatrix}$$

Where:

$\vec{\mathbf{X}}^{N}$: Position vector in NAD83 (CSRS)

$\vec{\mathbf{X}}^{I}_{96}$: Position vector in ITRF96

$\dot{\mathbf{R}}^{N}$: Time derivative of the rotation matrix (NNR-NUVEL-1A)

t_0 : Reference epoch (1997.0)

Through the link with the international system, the three translation parameters (T_X, T_Y, T_Z) confirm that the NAD83 terrestrial reference system is almost geocentric.

General transformation equation

Since ITRF96, the ITRF has evolved with the implementation of ITRF97, ITRF2000 and ITRF2005. With each new realization, the IERS determines the 7+7 parameters that link this realization to its predecessor (section 5.1.1). Introducing these parameters into equation (5) gives the following general mathematical expression for transforming any ITRF position to NAD83 (CSRS):

$$\vec{\mathbf{X}}^{\mathrm{N}} = \left[\vec{\mathbf{T}}(t_0) + \dot{\vec{\mathbf{T}}}(t - t_0)\right] + \left[1 + S(t_0) + \dot{S}(t - t_0)\right]\vec{\mathbf{X}}^{\mathrm{I}}_y(t) + \left[\mathbf{R}(t_0) + \dot{\mathbf{R}}(t - t_0)\right]\vec{\mathbf{X}}^{\mathrm{I}}_y(t) \qquad (6)$$

Where:

$\vec{\mathbf{X}}^{\mathrm{N}}$: Position vector in NAD83 (CSRS)

$\vec{\mathbf{X}}^{\mathrm{I}}_y$: Position vector in one of the ITRFy

$\dot{\vec{\mathbf{T}}}$: Time derivative of the translation vector $\vec{\mathbf{T}}$

$\dot{S}$: Time derivative of the scale factor S

$\dot{\mathbf{R}}$: Time derivative of the rotation matrix $\mathbf{R}$

t : Observation epoch

t_0 : Reference epoch (1997.0)

Figure 24 summarizes the links between an ITRF position at a given epoch (*t*) and NAD83 (CSRS) and vice versa. Contrary to what is written in the previous paragraph, the transformation parameters used between ITRF96 and ITRF97 are the IERS parameters. Canada and the US preferred to use the IGS parameters (Soler and Snay, 2004).

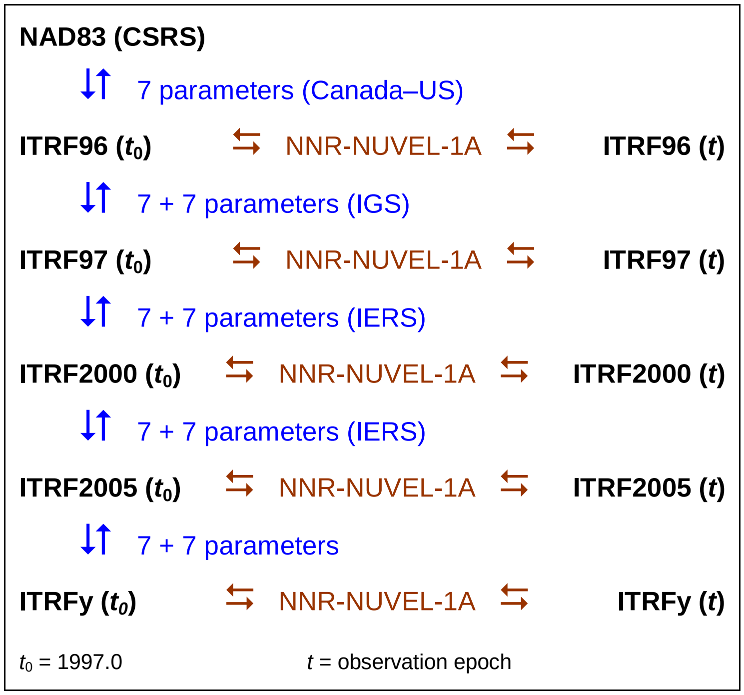


Figure 24: Links between NAD83 (CSRS) and the ITRFs

The values of the parameters in this transformation for ITRF2005 are presented in Table 12 (Craymer, 2006b). The rotation parameters from the NNR-NUVEL-1A model (Table 11) are included in the parameters $\dot{\varepsilon}_X$, $\dot{\varepsilon}_Y$ et $\dot{\varepsilon}_Z$ of the time derivative matrix $\dot{\mathbf{R}}$.

Table 12: ITRF2005 → NAD83 (CSRS) transformation parameters

Parameter	Value	Parameter	Value
$T_X(t_0)$	0.9963 m	$\dot{T}_X$	0.0005 m/yr
$T_Y(t_0)$	-1.9024 m	$\dot{T}_Y$	-0.0006 m/yr
$T_Z(t_0)$	-0.5219 m	$\dot{T}_Z$	-0.0013 m/yr
$\varepsilon_X(t_0)$	-25.915 m″	$\dot{\varepsilon}_X$	-0.067 m″/yr
$\varepsilon_Y(t_0)$	-9.426 m″	$\dot{\varepsilon}_Y$	0.757 m″/yr
$\varepsilon_Z(t_0)$	-11.599 m″	$\dot{\varepsilon}_Z$	0.051 m″/yr
$S(t_0)$	0.775 ppb	$\dot{S}$	-0.102 ppb/yr

m″: milliarcsecond
ppb: parts per billion (10^{-3} ppm)
t_0 = 1997.0

The GSD developed the TRNOBS software to help transform a NAD83 (CSRS) coordinate to one of the ITRF solutions in a given epoch and vice versa. TRNOBS also makes it possible to transform vectors in these different solutions. An online version of TRNOBS on the GSD Web site can transform coordinates (http://www.geod.nrcan.gc.ca/tools-outils/trnobs_e.php).

5.2.1.4 Evolution of NAD83 (CSRS)

NAD83 (CSRS) evolves over time just as the ITRF and WGS84 frames do. A new version is normally associated with recent spatial measurements of the entire CBN. The calculation process consists in determining the ITRFy coordinates at observation epoch (*t*) for all the CBN points and CACS stations and then transforming them to NAD83 (CSRS) according to equation (6) schematized in Figure 24. The procedure for calculating the ITRFy (*t*) coordinates is explained below.

Section 5.2.1.2 states that some CACS stations are part of the ITRF network. They therefore have an ITRF position at reference epoch (t_0) and a velocity vector ($\dot{\vec{\mathbf{X}}}$). If the observation epoch (*t*) is different from the reference epoch (t_0), it is necessary to determine the ITRF positions at the observation epoch. This position is obtained by:

$$\vec{\mathbf{X}}_y^I(t) = \vec{\mathbf{X}}_y^I(t_0) + \dot{\vec{\mathbf{X}}}_y^I(t - t_0) \tag{7}$$

The CACS stations in the ITRF network are used as control points for calculating the ITRFy (*t*) coordinates of the other CACS stations. Then, the spatial measurements done on the CBN are processed based on the ITRFy (*t*) coordinates of the CACS stations. Thus, the active CACS and passive CBN components are known in ITRFy (*t*) and can be transformed to NAD83 (CSRS).

Over the years, several versions of NAD83 (CSRS) have been created. The main characteristics of the known versions as of April 2010 are presented in Table 13, except for version 1. Based on the first transformation, version 1 had limited distribution in the geodetic community and the related documentation is also rare. Because of the involvement of active CACS and passive CBN components, GSD designates CACS-*v* and CBN-*v* as the solutions associated with version v of NAD83 (CSRS). Appendix B gives a brief description of these versions.

Table 13: Versions of NAD83 (CSRS)

Version	ITRF	Reference Epoch (t_0)	Observation Epoch (*t*)
2	ITRF96	1997.0	1997.0
3	ITRF97	1997.0	1997.0
4	ITRF2000	1997.0	2002.0
5	ITRF2005	2000.0	2006.0

GSD does not indicate the version of NAD83 (CSRS) that the coordinates it publishes refer to, but rather the observation epoch. For instance, epoch 1997.0 indicates that the coordinate comes from version 2, 3 or 3.1. Epoch 2002.0 indicates that the coordinate comes from version 4. For a point with epoch 2004.88, for example, this means that the point is not part of a current version. As for the DRG, because it uses the CBN-2 version, a note to this effect is entered on the data sheet: "(Coordinates based on the adjustment of the Canadian Base Network v2.0 March 1998)."

The calculation procedure used to obtain a new version of NAD83 (CSRS) introduces a planimetric bias and height variations when the observation epoch (t) differs from the reference epoch (t_0). In the calculations, the annual velocity can be managed using two different approaches to transform ITRFy (t) positions to NAD83 (CSRS). In the first approach, the velocity vector from ITRFy (t_0) can be used to position the CACS stations of the ITRF network at the observation epoch. In the second approach, the NNR-NUVEL-1A model can be used. The velocity vector from ITRFy (t_0) is three-dimensional (horizontal and vertical), whereas the NNR-NUVEL-1A models only the horizontal displacement. Thus, coming back to the reference epoch, only the horizontal displacement is taken into account. Vertical motion such as that caused by post-glacial rebound, is not taken into account. In addition, using the NNR-NUVEL-1A model introduces a bias into the procedure. This model overestimates the velocity of the tectonic plate by about 2 mm/yr (Craymer, 2006a) at our latitudes.

Example of variation

To present this situation numerically, the positions from the CBN-2, CBN-3, CBN-3.1 and CBN-4 versions for point 942005 of the CBN located at Kuujjuarapik are analyzed (Table 14). Table 15 gives the standard deviation of the position for each version. Figure 25 shows the difference between the CBN-2 and CBN-4 versions for the CBN points in Quebec and the Maritimes.

Table 14: Evolution of NAD83 (CSRS) versions at Kuujjuarapik

Version	Geodetic Coordinates			MTM Coordinates		
	Latitude	Longitude	Height	North	East	Zone
CBN-2	55° 16′ 42.06215″	77° 44′ 43.54252″	0.226 m	6,128,312.159 m	225,662.684 m	9
CBN-3	55° 16′ 42.06216″	77° 44′ 43.54255″	0.214 m	6,128,312.160 m	225,662.683 m	9
CBN-3.1	55° 16′ 42.06216″	77° 44′ 43.54252″	0.211 m	6,128,312.160 m	225,662.684 m	9
CBN-4	55° 16′ 42.06203″	77° 44′ 43.54218″	0.274 m	6,128,312.156 m	225,662.690 m	9

Table 15: Standard deviation of NAD83 (CSRS) versions at Kuujjuarapik

Version	Latitude	Longitude	Height
CBN-2	0.002 m	0.002 m	0.008 m
CBN-3	0.002 m	0.002 m	0.009 m
CBN-3.1	0.002 m	0.002 m	0.009 m
CBN-4	0.001 m	0.001 m	0.003 m

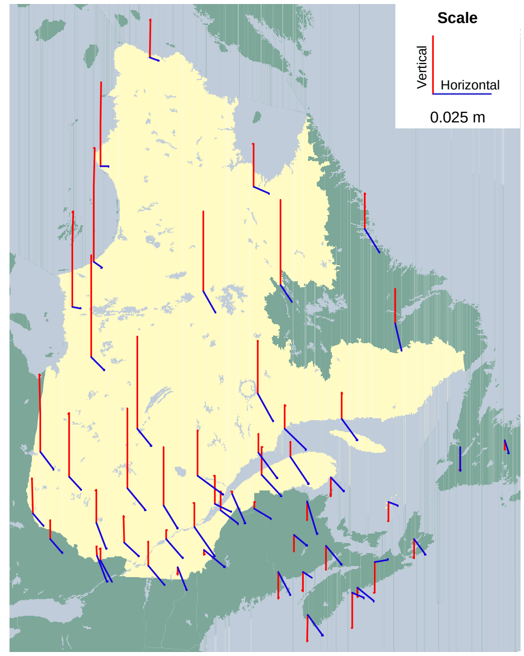


Figure 25: CBN-2 and CBN-4

Versions 2, 3 and 3.1 are identical from the planimetric aspect, but there is a slight discrepancy in height between version 2 and versions 3 and 3.1. Although they share the same observation epoch (1997.0), these small variations can be explained by the fact that these three versions are based on different ITRFs and additional data measured at different periods from 1996 to 2000.

Version 4 presents more significant discrepancies compared with the three previous versions. The main factors behind these discrepancies are a new observation epoch (2002.0) and physical variations due to terrestrial movement (post-glacial rebound). From the planimetric aspect, using the NNR-NUVEL-1A model would have introduced a bias of about 0.010 m, since there are five years separating versions 2, 3 and 3.1 ($t = 1997.0$) from version 4 ($t = 2002.0$). The difference of 0.007 m between versions 2 and 4 is this order of magnitude. From the height aspect, the difference is 0.048 m between versions 2 and 4, and 0.060 m between versions 3 and 4. Since the post-glacial rebound in this sector is about 11 mm/yr according to the ICE-4G model (Figure 5) and Figure 8, this sector would be lifted up by about 0.055 m in five years. The theoretical uplift corresponds to the observed uplift. Based on these explanations, the discrepancies in position in version 4 compared with previous versions are not due to errors. These discrepancies are due to different observation epochs and post-glacial rebound.

Quebec uses the CBN-2 version to position its geodetic network. The discrepancy between its coordinates and the new versions of NAD83 (CSRS) will increase as time passes. For instance, for observation epoch $t = 2007.0$, the planimetric bias will be about 0.020 m, whereas the height discrepancy could reach 0.1 m in the Hudson's Bay sector. The increase in the planimetric bias and height discrepancies over time demonstrates the importance of having an accurate model to follow the three-dimensional motion of the North American tectonic plate, if the current approach (Figure 24) persists.

5.2.2 NAD83 and WGS84

By definition, the NAD83 and WGS84 systems are compatible since they have the same characteristics and the GRS80 and WGS84 ellipsoids are (almost) identical. When it was created, NAD83 (1986) was therefore reputed to be compatible with the Doppler version of WGS84.

In reality, the NAD83 and WGS84 frames are not compatible. Today, WGS84 is geocentric, whereas NAD83 is not. Despite new versions of WGS84, the NGA states in Technical Report 8350.2 (NIMA, 1997) that there is no translation between WGS84 and NAD83. Figure 26 shows the difference between the recent realizations of the WGS84 and NAD83 systems (Craymer, 2006a). In Quebec, the horizontal offset is about 1.5 m, whereas the vertical offset can reach 1.5 m. The values of this offset also apply to ITRS realizations. An example of the difference is presented in section 5.3.4.

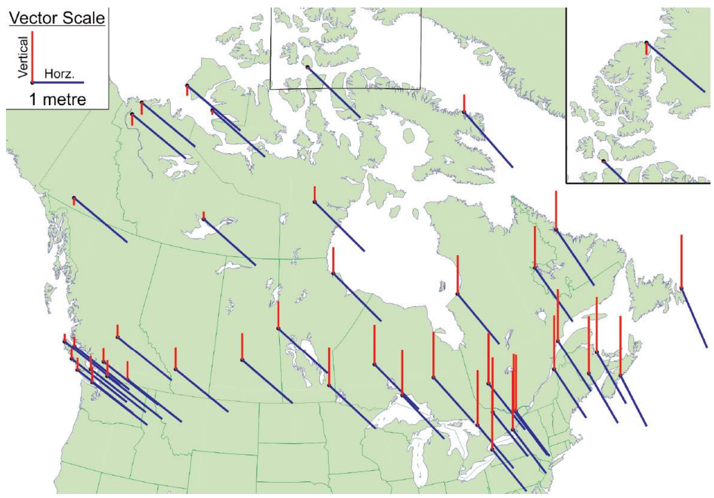


Figure 26: Difference between NAD83 and WGS84

Most GPS makers use the parameters published by the NGA to transform a WGS84 position to another datum. Therefore, when users choose NAD83 as the reference system, no correction is applied to WGS84 positions.

5.3 GPS Calculation

The purpose of this section is to demonstrate the influence of terrestrial reference systems in calculating a position from GPS observations. Before getting into the topic, the positioning techniques and methods are briefly reviewed. The descriptions are succinct but do not go into the technical details. Excellent documents and Internet sites provide detailed descriptions of these positioning techniques and methods.

There are two techniques for determining a position from GPS observations: absolute positioning and relative positioning. For each technique, the positioning method can be in real time, which means that a position is obtained instantaneously, or by post-processing, which means that a position is obtained once the observations have been completed. The choice of technique and method depends on the work to be done. For instance, in navigation, it is necessary to know one's position immediately, whereas in geodetics, determining a very accurate position is not possible until after the observations.

5.3.1 Absolute Positioning

Absolute positioning, also known as single point positioning, requires the use of a single GPS receiver. Its position is determined in relation to the origin of the terrestrial reference frame (Figure 27) used to calculate satellite positions. In the real-time method, the receiver's position is calculated using the ephemeris transmitted by each GPS satellite. These ephemeris contain all the values to calculate the position of the satellite in WGS84 at each observation epoch. Thus, the positions obtained in real time are in the WGS84 frame. It is possible to do absolute positioning in post-processing.

As with real time, calculation in post-processing requires known satellite positions. This position can be computed from broadcast or "precise" ephemeris. "Precise" ephemeris are post-processed satellite positions and clock corrections computed by a few organizations, such as GSD.

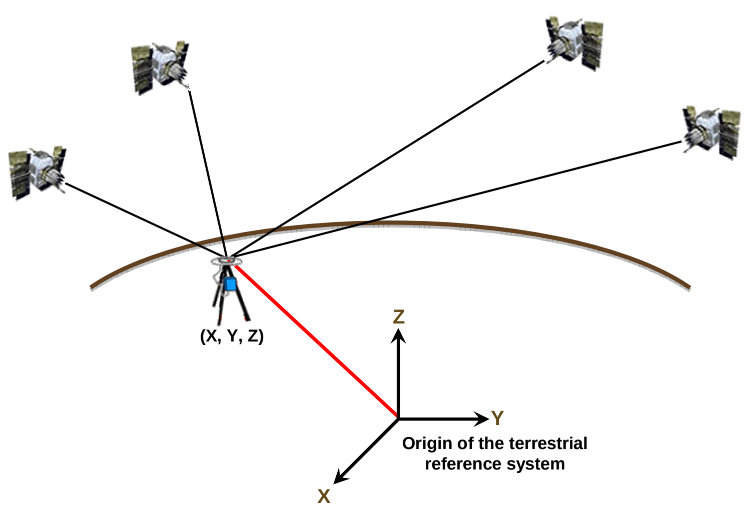


Figure 27: Absolute positioning

Precise ephemeris files are text files structured according to the SP3 format developed by the National Geodetic Survey (NGS) of the National Oceanic and Atmospheric Administration (NOAA — Department of Commerce) in the US. They contain the position (X, Y, Z) of the satellites in a specific frame, along with clock corrections. In general, they are daily files (from 0:00 to 23:45, GPS time) containing data every 15 minutes.

The main producer of combined precise ephemeris is the IGS. Its mission is to provide the highest quality data and products as the standard for GNSS in support of Earth science research, multidisciplinary applications and education. To accomplish this mission, a network of tracking stations (Figure 28) (IGS, 2009) was implemented, along with a frame to manage and process the data. Figure 29 (IGS, 2009) shows the network of stations in North America.

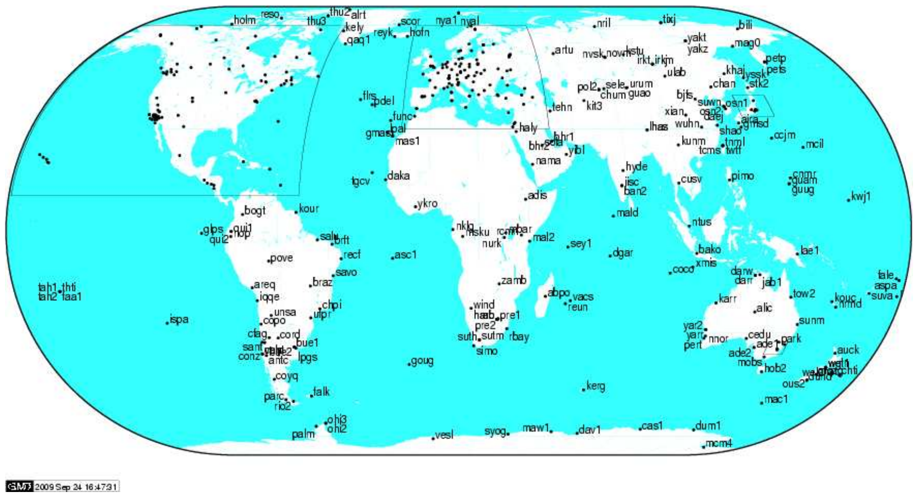


Figure 28: IGS network of tracking stations

Figure 29: IGS network of tracking stations in North America

The main products offered by the IGS are:

- Precise GPS orbits;
- GPS satellite and tracking station clock information;

- IGS tracking station position and velocity.

The IGS calculates three types of orbit solutions and clock corrections in order to meet different user needs:

- Final: Final products are available from 12 to 18 days later and contain 24 hours of information.
- Rapid: Rapid products are available at 17:00 (UT) the next day and contain 24 hours of information.
- Ultra-Rapid: Ultra-rapid products are published four times daily (at 03:00, 09:00, 15:00 and 21:00 UT) and contain 48 hours of information. The first half is calculated from observations and the second half is predicted.

For its own purposes, the IGS realizes a reference frame equivalent to the last ITRF version. The IGS orbits are calculated in their datum. Table 16 summarizes the evolution of the datums used by the IGS over the past few years. The “Date” and “GPS Week” columns correspond to the date this frame was deployed for calculating IGS products. Therefore, reference frame IGS05, aligned to ITRF2005, has been in effect since November 5, 2006. This date corresponds to the start of GPS week 1400.

Table 16: IGS reference frame

Name	Date	GPS Week	Aligned to
IGS05	2006-11-05	1400	ITRF2005
IGb00	2004-01-11	1253	ITRF2000
IGS00	2001-12-02	1143	ITRF2000
IGS97	2000-06-04	1065	ITRF97
ITRF97	1999-08-01	1021	----

GSD also calculates precise GPS ephemeris. Unlike the IGS, GSD also provides ephemeris in NAD83 (CSRS). Like the IGS, GSD produces rapid and final ephemeris, and they are available at the same times.

It is possible to obtain precise GPS ephemeris in the NGA’s WGS84 (G1150) frame.

Using WGS84 or IGS05 (ITRF2005) in post-processing gives the position at the observation epoch. Comparing this position with the one obtained in NAD83 (CSRS) would show the combined effect of the offset of its origin in relation to these geocentric datums, the motion of the North American tectonic plate, and post-glacial rebound. Using the ephemeris in NAD83 (CSRS) would eliminate these biases, except for the post-glacial rebound and the discrepancies between the different versions of NAD83 (CSRS). Numerical examples are presented later on.

To summarize, regardless of whether absolute positioning is done in real time or in post-processing, the position of the GPS receiver is always associated with the datum for the orbits.

5.3.2 Relative Positioning

Relative positioning requires the use of two GPS receivers simultaneously. One of the receivers has a known position (fixed receiver) in a global or regional datum. In this technique, the position of the second receiver (mobile receiver) is determined in relation to the position of the fixed receiver. The method consists in determining the spatial components (ΔX, ΔY, ΔZ) of the vector between the two receivers in real time or in post-processing (Figure 30).

The vector between the two receivers is calculated in the datum used for the satellite orbits. The datum for the fixed receiver must be identical to the datum used for the orbits. Thus, if the datum for the fixed points is ITRF2005, the vectors can be calculated with the broadcast ephemeris (WGS84) or precise ephemeris (WGS84, IGS05) since these datums are aligned to ITRF2005 to centimetre accuracy. In this case, the coordinates for the fixed points must be from the observation epoch. GSD uses this approach to calculate the coordinates for its networks in a global datum before transforming them to NAD83 (CSRS) (section 5.2.1.3).

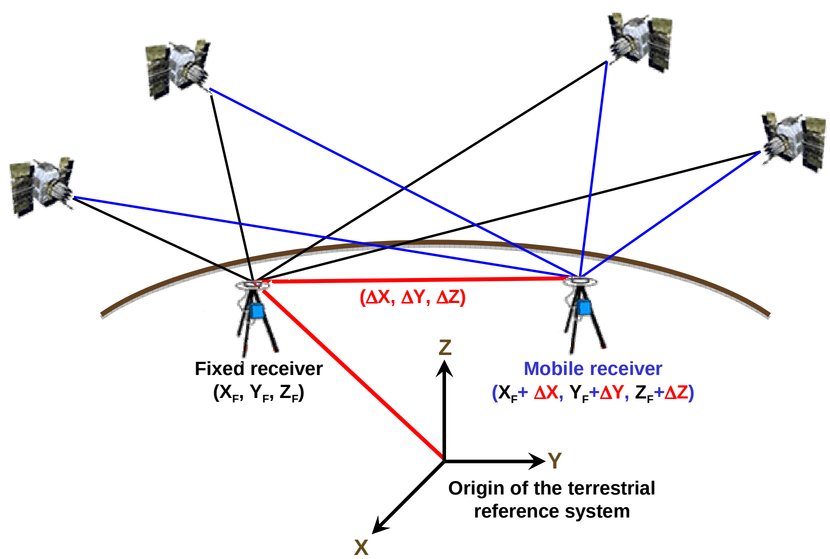


Figure 30: Relative positioning

When the datum for the fixed receiver is not identical to the datum used for the orbits, the position of the mobile receiver can be affected. For instance, if the datum for the fixed points is NAD83, using the IGS05 orbit datum instead of the NAD83 datum introduces a bias in the vectors. Several solutions are available for users to correct this situation.

Since IGS05 is aligned to ITRF2005 to centimetre accuracy, it is possible to use the ITRF2005 — NAD83 (CSRS) transformation to transform a vector calculated in the IGS05 datum to the NAD83 datum. The mathematical expression for this transformation is:

$$\Delta\vec{\mathbf{X}}^{\mathrm{N}}(t) = \left[1 + S(t_0) + \dot{S}(t - t_0)\right]\Delta\vec{\mathbf{X}}^{\mathrm{I}}_{2005}(t) + \left[\mathbf{R}(t_0) + \dot{\mathbf{R}}(t - t_0)\right]\Delta\vec{\mathbf{X}}^{\mathrm{I}}_{2005}(t) \tag{8}$$

$$\Delta\vec{\mathbf{X}} = \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}$$

Where $\Delta\vec{\mathbf{X}}^{\mathrm{N}}$ and $\Delta\vec{\mathbf{X}}^{\mathrm{I}}_{2005}$ are the position difference vectors in NAD83 (CSRS) and ITRF2005, respectively.

This transformation corrects the bias of about 0.1 ppm caused mainly by a rotation of the vector. The larger the vector, the larger the bias if no transformation is applied. An example of bias is given in section 5.3.5.

A more rigorous approach would be to transform the NAD83 (CSRS) position of the fixed point in ITRF2005 at the observation epoch, calculate the vectors using the IGS's precise ephemeris, and transform this position to NAD83 (CSRS). The simplest approach is to use GSD's NAD83 ephemeris when calculating the vectors when the datum for the fixed points is NAD83.

Contrary to absolute positioning, it is almost impossible to calculate positions in the WGS84 datum in relative positioning. In order to do so, it is necessary to have a point whose coordinates are known in WGS84 (G1150). But, civilian users do not have access to the GPS data for stations in the GPS tracking network, even if their coordinates are known.

In the real-time method, the orbits are calculated with the broadcast ephemeris (WGS84). The position of the fixed receiver can be in ITRF or NAD83. In this method, the vectors are generally relatively short (smaller than 20 km) and the bias produced by using different datums is less than 2 mm. The effect of this bias on the position of the mobile receiver is practically null.

To summarize, regardless of whether relative positioning is done in real time or in post-processing, the position of the GPS receiver is always associated with the datum for the reference point. However, this position can be influenced by a relatively large bias if the datum for the orbits is different from the datum for the fixed point.

5.3.3 Online Applications

Before giving examples illustrating the different concepts covered so far in this chapter, this section presents the applications that are accessible online over the Internet. The results of one of these applications are used to illustrate these concepts.

These online applications allow calculation of the position of a point in a GPS data file in RINEX format. Processing is optimized for geodetic receivers (dual frequency). The calculations are done with the precise ephemeris available when requesting processing. These applications are offered by various government geodetic organizations or various research centres.

GSD introduced PPP (precise point positioning) for absolute positioning (Mireault et al., 2008). The calculations are done in IGS05 used by the IGS (assuming the orbits were computed in that frame). Since IGS05 is aligned precisely to ITRF2005, the PPP result is considered to be a position in ITRF2005. This position is then transformed to NAD83 (CSRS). In the transformation process to NAD83 (CSRS), GSD introduced a model that takes into account the distortions caused by the NNR-NUVEL-1A model and post-glacial rebound. With this distortion model, users can choose to have their NAD83 (CSRS) position at the observation epoch, at epoch 2002.0 for version 4 and at epoch 1997.0 for versions 2, 3 and 3.1.

NGS introduced OPUS (Online Positioning User Service) for relative positioning (www.ngs.noaa.gov/OPUS/). The calculations are done in ITRF2000 based on the closest three stations in the CORS (Continuously Operating Reference Station) network with the IGS's available ephemeris (ultra-rapid, rapid, final). This position is transformed to NAD83 (CORS96), and is compatible with version 4 of NAD83 (CSRS).

Other services similar to OPUS are available. For instance, AUSPOS (Online GPS Processing Service) for relative positioning (www.ga.gov.au/geodesy/sgc/wwwgps/) offered by the Australian government's Geoscience Australia. The calculations are done in ITRF2000 with the IGS's available ephemeris (ultra-rapid, rapid, final). Another relative positioning service is SCOUT (Scripps Coordinate Update Tool) from the Scripps Orbit and Permanent Array Center (SOPAC) (http://sopac.ucsd.edu/cgi-bin/SCOUT.cgi). The calculations are done in ITRF2005 with the IGS's available ephemeris (ultra-rapid, rapid, final).

For all these online services, the position in the ITRF datum is at the observation epoch.

5.3.4 Positions in Context

To highlight the different concepts described in this chapter, the data from two CBN points converted into CACS stations are used. They are for station 962002 (LPOC) at Sainte-Anne-de-la-Pocatière, and station 932051 (VALD) at Val-d'Or. The GPS data observed on April 1, 2008 (*t* = 2008.25) by these two stations were retrieved on GSD's website. They are the LPOC0920.08O and VALD0920.08O RINEX files that contain 24 hours of data. For the purposes of the comparison, these two files were sent to the online PPP service to obtain an ITRF2005 position and a NAD83 (CSRS) position at the observation epoch.

Tables 17 and 18 present the NAD83 (CSRS) position of the LPOC and VALD stations in the CBN-2, CBN-3, CBN-3.1 and CBN-4 versions, as well as the position from PPP. The PPP application estimated the accuracy of the NAD83 (CSRS) positions to 0.003 m in latitude, 0.005 m in longitude and 0.010 m in height.

Table 17: NAD83 (CSRS) positions at LPOC

Version	Geodetic Coordinates			MTM Coordinates (Zone 7)	
	Latitude	Longitude	Height	North	East
CBN-2	47° 20′ 28.98074″	70° 0′ 30.79594″	104.291 m	5,244,792.642 m	341,934.891 m
CBN-3	47° 20′ 28.98077″	70° 0′ 30.79609″	104.284 m	5,244,792.643 m	341,934.888 m
CBN-3.1	47° 20′ 28.98081″	70° 0′ 30.79611″	104.281 m	5,244,792.644 m	341,934.888 m
CBN-4	47° 20′ 28.98054″	70° 0′ 30.79542″	104.298 m	5,244,792.636 m	341,934.902 m
PPP	47° 20′ 28.9801″	70° 0′ 30.7948″	104.315 m	5,244,792.622 m	341,934.915 m

Table 18: NAD83 (CSRS) positions at VALD

Version	Geodetic Coordinates			MTM Coordinates (Zone 9)	
	Latitude	Longitude	Height	North	East
CBN-2	48° 5′ 49.37144″	77° 33′ 50.98064″	313.745 m	5,329,233.350 m	225,544.054 m
CBN-3	48° 5′ 49.37147″	77° 33′ 50.98077″	313.738 m	5,329,233.352 m	225,544.051 m
CBN-3.1	48° 5′ 49.37147″	77° 33′ 50.98080″	313.735 m	5,329,233.352 m	225,544.051 m
CBN-4	48° 5′ 49.37120″	77° 33′ 50.98027″	313.772 m	5,329,233.343 m	225,544.062 m
PPP	48° 5′ 49.3711″	77° 33′ 50.9791″	313.842 m	5,329,233.340 m	225,544.086 m

As mentioned in section 5.2.1.4, there is practically no difference between the CBN-2, CBN-3 and CBN-3.1 versions. Tables 17 and 18 confirm this. The average position of these versions is used to simplify analysis of the concepts (Table 19).

Table 19: Average position of versions 2, 3 and 3.1

Station	North	East	Height
LPOC	5,244,792.643 m	341,934.889 m	104.285 m
VALD	5,329,233.351 m	225,544.052 m	313.739 m

The analysis of the variations of the position of point 942005 (Kuujjuarapik) in section 5.2.1.4 applies here. Table 20 shows the difference (ΔN, ΔE and Δh) between the NAD83 (CSRS) average position (Table 19) and the CBN-4 and PPP positions (Tables 17 and 18) for the LPOC and VALD stations. This table also shows the annual variation of this difference ($\Delta N/\Delta t$, $\Delta E/\Delta t$, $\Delta h/\Delta t$).

Table 20: Differences from average NAD83 (CSRS) positions

	ΔN (m)	ΔE (m)	Δh (m)	ΔN/Δ*t* (m/yr)	ΔE/Δ*t* (m/yr)	Δh/Δ*t* (m/yr)
LPOC						
Version 4 (Δ*t* = 5 years)	-0.007	0.013	0.013	-0.001	0.003	0.003
Version PPP (Δ*t* = 11.25 years)	-0.021	0.026	0.030	-0.002	0.002	0.003
VALD						
Version 4 (Δ*t* = 5 years)	-0.008	0.010	0.032	-0.002	0.002	0.006
Version PPP (Δ*t* = 11.25 years)	-0.011	0.034	0.103	-0.001	0.003	0.009

The planimetric position difference and its annual variation correspond closely in terms of direction and magnitude to the velocity overestimate of the North American plate in the NNR-NUVEL-1A model. The height position difference and its annual variation are not the same at each station. They are located in different uplift zones caused by the post-glacial rebound. The variation measured with PPP alone cannot confirm an uplift at the LPOC station, because it is too small. Recall that the heights calculated with GPS measurements are much less accurate than the planimetric components. At the VALD station, the variation measured with PPP is large enough to be greater than its inaccuracy and thereby shows that there is significant uplift. Figure 10 confirms this fact.

Tables 21 and 22 show the transformation of the NAD83 (CSRS) position, using the CBN-2 version, to the ITRF96, ITRF97, ITRF2000 and ITRF2005 position at epoch 1997.0 and the ITRF2005 position at epoch 2008.25 for the LPOC and VALD stations, respectively. The ITRF2005 position from the PPP is also given in these tables.

Table 21: ITRF positions at LPOC

Version	Geodetic Coordinates			MTM Coordinates (Zone 7)	
	Latitude	Longitude	Height	North	East
ITRF96 (1997.0)	47° 20′ 29.01522″	70° 0′ 30.79731″	103.221 m	5,244,793.706 m	341,934.856 m
ITRF97 (1997.0)	47° 20′ 29.01495″	70° 0′ 30.79754″	103.220 m	5,244,793.698 m	341,934.851 m
ITRF2000 (1997.0)	47° 20′ 29.01527″	70° 0′ 30.79794″	103.226 m	5,244,793.708 m	341,934.842 m
ITRF2005 (1997.0)	47° 20′ 29.01531″	70° 0′ 30.79796″	103.225 m	5,244,793.709 m	341,934.842 m
ITRF2005 (2008.25)	47° 20′ 29.01805″	70° 0′ 30.80758″	103.238 m	5,244,793.792 m	341,934.640 m
ITRF2005 (PPP)	47° 20′ 29.0174″	70° 0′ 30.8065″	103.262 m	5,244,793.773 m	341,934.662 m

Table 22: ITRF positions at VALD

Version	Geodetic Coordinates			MTM Coordinates (Zone 9)	
	Latitude	Longitude	Height	North	East
ITRF96 (1997.0)	48° 5′ 49.40530″	77° 33′ 50.99162″	312.737 m	5,329,234.399 m	225,543.841 m
ITRF97 (1997.0)	48° 5′ 49.40501″	77° 33′ 50.99186″	312.736 m	5,329,234.391 m	225,543.836 m
ITRF2000 (1997.0)	48° 5′ 49.40530″	77° 33′ 50.99224″	312.743 m	5,329,234.400 m	225,543.829 m
ITRF2005 (1997.0)	48° 5′ 49.40534″	77° 33′ 50.99226″	312.741 m	5,329,234.401 m	225,543.828 m
ITRF2005 (2008.25)	48° 5′ 49.40695″	77° 33′ 51.00241″	312.755 m	5,329,234.454 m	225,543.619 m
ITRF2005 (PPP)	48° 5′ 49.4066″	77° 33′ 51.0008″	312.852 m	5,329,234.442 m	225,543.652 m

To show the variations between all these positions, Table 23 gives the difference (ΔN, ΔE and Δh) between the ITRF96 (1997.0) position and the other versions of the ITRF and PPP (Tables 21 and 22) for the LPOC and VALD stations. This table also shows the annual variation at epoch 2008.25 (ΔN/Δ*t*, ΔE/Δ*t*, Δh/Δ*t*).

Table 23: Differences from ITRF96 (1997.0) positions

	ΔN (m)	ΔE (m)	Δh (m)	ΔN/Δ*t* (m/yr)	ΔE/Δ*t* (m/yr)	Δh/Δ*t* (m/yr)
LPOC						
ITRF97 (1997.0)	-0.008	-0.005	-0.001			
ITRF2000 (1997.0)	0.002	-0.014	0.005			
ITRF2005 (1997.0)	0.003	-0.014	0.003			
ITRF2005 (2008.25) (Δ*t* = 11.25 years)	0.086	-0.216	0.017	0.008	-0.019	0.001
ITRF2005 (PPP) (Δ*t* = 11.25 years)	0.067	-0.194	0.041	0.006	-0.017	0.004
VALD						
ITRF97 (1997.0)	-0.008	-0.005	-0.001			
ITRF2000 (1997.0)	0.001	-0.012	0.006			
ITRF2005 (1997.0)	0.002	-0.013	0.004			
ITRF2005 (2008.25) (Δ*t* = 11.25 years)	0.055	-0.222	0.018	0.005	-0.020	0.002
ITRF2005 (PPP) (Δ*t* = 11.25 years)	0.043	-0.189	0.114	0.004	-0.017	0.010

For epoch 1997.0, the variations show mainly the differences between the ITRF solutions, except for the ITRF2000 and ITRF2005 solutions that give approximately the same positions. For the ITRF2005 (2008.25) position of the two stations, the planimetric difference shows the displacement of the North American tectonic plate in 11.25 years, or an annual displacement of about 0.02 m/yr. The slight height difference comes from the transformation process (Figure 24) that does not take vertical motion into account. For the ITRF2005 (PPP) position of the two stations, the planimetric difference also shows the displacement of the North American tectonic plate. However, the height difference, especially at the VALD station, is caused by post-glacial rebound. The annual variation of the ITRF2005 (PPP) position at the VALD station matches the variation presented in Table 1.

Table 24 shows the difference between the ITRF2005 (2008.25) and ITRF2005 (PPP) position at both stations. The observations made subsequent to the analysis of the position from PPP presented in Table 20 apply to Table 24. The planimetric variations are caused mainly by the deficiencies of the NNR-NUVEL-1A model, whereas the height variations are due to the absence of a model for the post-glacial rebound.

Table 24: Difference between ITRF2005 (2008.25) and PPP (2008.25)

Station	ΔN (m)	ΔE (m)	Δh (m)	ΔN/Δ*t* (m/yr)	ΔE/Δ*t* (m/yr)	Δh/Δ*t* (m/yr)
LPOC	-0.019	0.022	0.024	-0.002	0.002	0.002
VALD	-0.012	0.033	0.097	-0.001	0.003	0.009

Table 25 shows the difference between the NAD83 (CSRS) average position (Table 19) and the ITRF96 (1997.0) and ITRF2005 (PPP) positions. The difference with ITRF96 (1997.0) shows the discrepancy between the two frames at epoch 1997.0, whereas the difference with ITRF2005 (PPP) shows the discrepancy between the two frames at two different epochs. During the 11.25 years, the tectonic plate continued to move and the isostatic equilibrium (post-glacial rebound) continued its process. Since PPP gives a position at the observation epoch, it is normal for it to evolve at the same rate as the displacement of the tectonic plate, including the internal phenomena such as isostatic equilibrium, on which the point is located. This demonstrates the importance of taking the epoch of the position into account when comparisons must be made.

Table 25: Difference between NAD83 (CSRS) and ITRF

	ΔN (m)	ΔE (m)	Δh (m)
LPOC			
ITRF96 (1997.0)	1.063	-0.033	-1.064
ITRF2005 (PPP)	1.130	-0.227	-1.023
VALD			
ITRF96 (1997.0)	1.048	-0.211	-1.002
ITRF2005 (PPP)	1.091	-0.400	-0.888

5.3.5 Vectors in Context

To highlight the different concepts described in this chapter and their effect on vectors, vectors from ITRF2005 (2008.25) are transformed to NAD83 (CSRS) with equation (8). The analysis involves three vectors of 100, 250 and 500 km oriented either north or east. The point of origin for these six vectors is the LPOC station. This section is not intended to provide a detailed analysis of the effect of the transformation, but to give it an order of magnitude.

Table 26 shows the differences (δΔX, δΔY, δΔZ) and (ΔN, ΔE, Δh) between the two datums for the 100, 250 and 500 km vectors oriented north. Table 27 presents the differences for the 100, 250 and 500 km vectors oriented east.

Table 26: South-north vectors: ITRF2005 (2008.25) → NAD83 (CSRS)

100 km			
ITRF2005 (2008.25): ΔX, ΔY, ΔZ	-25,323.199 m	69,607.252 m	67,183.987 m
NAD83 (CSRS): ΔX, ΔY, ΔZ	-25,323.196 m	69,607.262 m	67,183.978 m
δΔX, δΔY, δΔZ	0.003 m	0.010 m	-0.009 m
ΔN, ΔE, Δh	0.000 m	0.007 m	-0.012 m
250 km			
ITRF2005 (2008.25): ΔX, ΔY, ΔZ	-63,976.109 m	175,854.603 m	165,759.595 m
NAD83 (CSRS): ΔX, ΔY, ΔZ	-63,976.100 m	175,854.628 m	165,759.572 m
δΔX, δΔY, δΔZ	0.009 m	0.025 m	-0.023 m
ΔN, ΔE, Δh	0.001 m	0.017 m	-0.031 m
500 km			
ITRF2005 (2008.25): ΔX, ΔY, ΔZ	-130,125.644 m	357,683.422 m	324,052.211 m
NAD83 (CSRS): ΔX, ΔY, ΔZ	-130,125.627 m	357,683.470 m	324,052.164 m
δΔX, δΔY, δΔZ	0.017 m	0.048 m	-0.047 m
ΔN, ΔE, Δh	0.002 m	0.033 m	-0.061 m

Table 27: West-east vectors: ITRF2005 (2008.25) → NAD83 (CSRS)

100 km			
ITRF2005 (2008.25): ΔX, ΔY, ΔZ	93,563.808 m	35,267.169 m	0.000 m
NAD83 (CSRS): ΔX, ΔY, ΔZ	93,563.810 m	35,267.164 m	-0.004 m
δΔX, δΔY, δΔZ	0.002 m	-0.005 m	-0.004 m
ΔN, ΔE, Δh	-0.007 m	0.000 m	0.001 m
250 km			
ITRF2005 (2008.25): ΔX, ΔY, ΔZ	232,204.615 m	92,145.199 m	0.000 m
NAD83 (CSRS): ΔX, ΔY, ΔZ	232,204.620 m	92,145.186 m	-0.011 m
δΔX, δΔY, δΔZ	0.005 m	-0.013 m	-0.011 m
ΔN, ΔE, Δh	-0.017 m	0.000 m	0.001 m
500 km			
ITRF2005 (2008.25): ΔX, ΔY, ΔZ	457,908.015 m	197,127.895 m	0.000 m
NAD83 (CSRS): ΔX, ΔY, ΔZ	457,908.025 m	197,127.870 m	-0.023 m
δΔX, δΔY, δΔZ	0.010 m	-0.025 m	-0.023 m
ΔN, ΔE, Δh	-0.035 m	-0.002 m	0.001 m

As mentioned in section 5.3.2, the larger the vector, the greater the bias (difference). Thus, for a vector of 500 km oriented north, using the IGS05 or WGS84 datum for calculating the vector can introduce a planimetric bias of 3 cm and a height bias of 6 cm if the point of origin is in the NAD83 datum.

For all GPS work over long distances in the NAD83 datum, i.e., more than 100 km, it is strongly recommended, and even mandatory, to use the precise ephemeris calculated in NAD83 by GSD. After this distance, the bias starts to equal the precision that can theoretically be obtained for a vector and thereby influence the accuracy of the mobile receiver's position. By extrapolation, small vectors (< 20 km), generally used in the real-time method, are hardly influenced by the use of different datums.

6 Vertical Datums

Because of its specific characteristics, the vertical datum evolves in a totally different world from the geodetic datum. For the geodetic datum, a simple mathematical surface such as an ellipsoid is used to determine the position of an object in space. The most complex part is situating this ellipsoid in space. For the vertical datum, the reference surface is a result of the Earth's gravitational field. This surface is called the geoid and it represents the equipotential surface that better defines the shape of the Earth.

6.1 Reference Surface

In his work titled *Géodésie générale*, Levallois (1969, p. 20) wrote [translation]:

> The fundamental phenomenon of physics that determines the shape of the Earth is gravity.
>
> We know that gravity is the resultant attracting force that is applied to every material point, on the one hand by the Newtonian set of forces of attraction of other points on the Earth, and on the other by the centrifugal force due to the Earth's rotation about its axis.
>
> Under the force of gravity, a plumb line in its resting position is in equilibrium along the physical vertical, this is the very definition of the vertical at a point.

Because of these forces, gravity is perpendicular to the surface of a quiescent fluid at every point (Figure 31). Such a surface is called a level surface, or more precisely, an "equipotential surface."

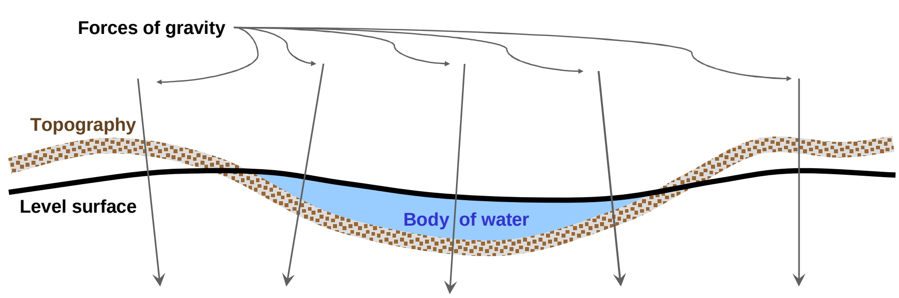


Figure 31: Gravity and level surface

On this surface, the gravity potential (W), or geopotential, is constant and is expressed in m^2/s^2. Thus, the potential difference between two surfaces along the vertical is written (Meyer et al., 2006):

$$dW = -g\,dH \tag{9}$$

Where H is the distance in metres separating the two surfaces at a given point and g is the gravity in m/s^2. The gravity corresponds to the force of gravity $|\vec{g}|$. A potential difference of 1 m^2/s^2 represents a difference of about 0.1 m between two surfaces, using an average value of 9.8 m/s^2 for the gravity.

The number of equipotential surfaces is infinite. Their main characteristics are that they are not parallel and do not intersect. These surfaces consist of bumps and hollows depending on the gravity, which varies at every point. Among other things, the gravity depends on the latitude, height and distribution of the variable density masses under the ground. Figure 32 shows the shape of equipotential surfaces near a dense mass (D) and a light mass (L). With its greater attracting potential, the dense mass draws the equipotential surfaces closer compared with the light mass. According to equation (9), for the same potential difference (dW) at points A and B, the gravity at point A is greater than the gravity at point B because the distance separating surfaces W_0 and W_N is shorter.

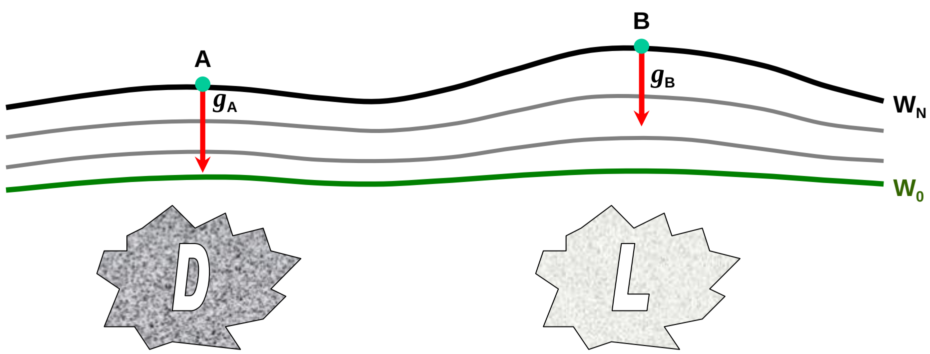


Figure 32: Equipotential surfaces

There is one specific equipotential surface. A priori, the oceans appear to represent an equipotential surface because they form a continuous surface. That is why the oceans have been used as a reference surface for heights over the centuries. The equipotential surface that passes closest to the surface of the oceans at rest (absence of tides or currents, among other things) is called the geoid (W_0). This surface is continuous despite the presence of the continents. Thus, equation (9) can be written as follows for a point A located above the geoid:

$$W_0 - W_A = \int_0^{H_A} g\,dH \tag{10}$$

Or more simply by:

$$C_A = \int_0^{H_A} g \, dH \tag{11}$$

Where C is the geopotential number that represents the potential difference with respect to the geoid at a specific point. H is the orthometric height, or the distance along the vertical between the two surfaces. Since *g* is not constant along the vertical between two equipotential surfaces, equation (11) can be simplified using the average value of the gravity ($\bar{g}$) to obtain:

$$C_A = \bar{g}H \tag{12}$$

Since the gravity varies at all points, this equation implies that, for a constant geopotential number, the orthometric height (H) varies continuously.

There is a convergence of equipotential surfaces towards the poles, caused by the difference in gravity that exists between the equator and the poles. For the same geopotential number at the equator and the poles, equation (12) gives a distance H that is greater at the equator than at the poles, because gravity is greater at the poles (9.83 m/s^2) than at the equator (9.78 m/s^2). Bowie and Avers (1914) mention that a distance of 1,000 m between two equipotential surfaces at the equator would give a distance of about 995 m at the poles between the same two surfaces.

This convergence creates interpretation problems. For example, a large body of water at rest with a south-north orientation appears level (Figure 33). By definition, this body of water has the same geopotential number for its entire surface. Since the water is not flowing out, the difference in orthometric height should be zero between its two extremities. However, if this lake is long enough to have a $\bar{g}$ that is smaller at the south end than the north, then $H_{South} > H_{North}$. Inversely, the same orthometric height at both ends of this lake would make it flow south, because the north end of the lake is on a greater equipotential surface compared with the south ($C_{South} < C_{North}$). Heiskanen and Moritz (1967, p. 172) summarize this situation as follows:

> The orthometric height differs for points of the same level surface because the level surfaces are not parallel. This gives rise to the well-known paradoxes of “water flowing uphill,” etc.

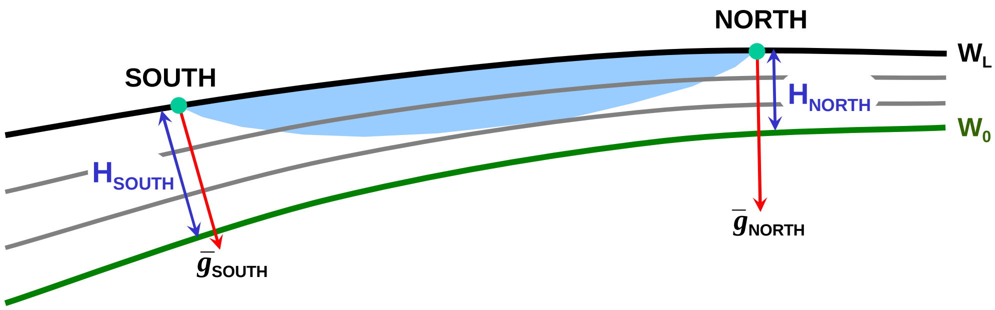


Figure 33: Convergence of equipotential surfaces

A height exists that is constant along an equipotential surface. In equation (12), the gravity varies continuously. However, the geopotential number is constant over the same equipotential surface. By dividing the geopotential number by a constant value of gravity, the result is a constant height on this surface. This height is called the “dynamic height” (H^D). The equation below expresses this relation:

$$C_A = \gamma_\phi H^D \qquad (13)$$

Where γ_ϕ is the normal gravity at a determined latitude. Normal gravity γ, or theoretical gravity, is a result of a mathematical equipotential surface (U). It is the surface of the ellipsoid. When defining an ellipsoid, several constants associated with its normal potential are defined. Normal gravity is calculated from a simple equation where the only variable is latitude. The result of this equation is a constant gravity along a parallel.

Dynamic height is generally used for managing large bodies of water. For instance, to manage the water levels in the Great Lakes and St. Lawrence Seaway, Canada and the US implemented the International Great Lakes Datum (IGLD) in 1985. This system calculates the dynamic heights with normal gravity at the 45th parallel. In the example in Figure 33, the dynamic height is constant over the entire surface of the lake, whereas the orthometric height varies at each point.

6.2 Levelling

Spirit levelling is a very accurate method for determining the height of a point in relation to a reference point. The height difference measured represents the difference in elevation between two points. Convergence to the north and especially the non-parallelism of equipotential surfaces influence levelling measurements. Figure 34 summarizes these effects by showing two different routes to determine the height of a point. Points A and D are located on the same equipotential surface. They therefore have the same equipotential number (C_A = C_D), but different orthometric heights ($H_A \neq H_D$).

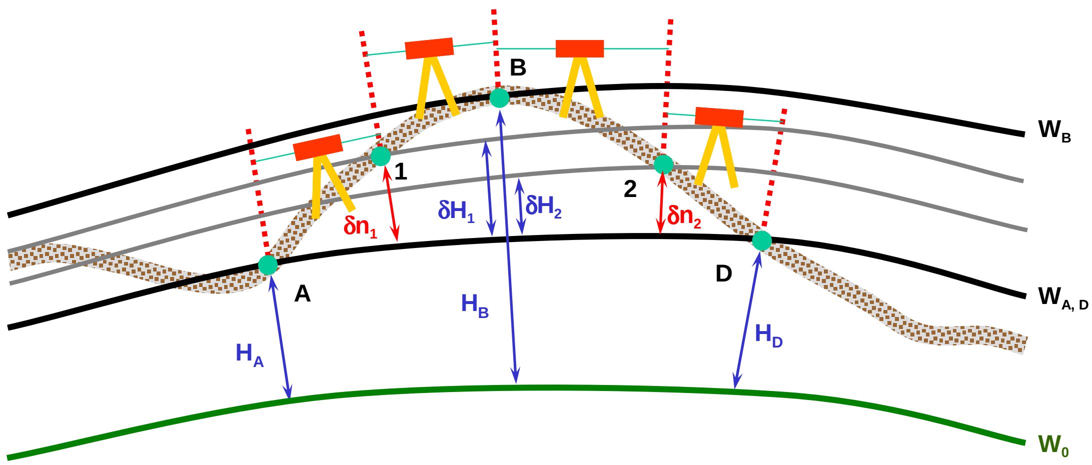


Figure 34: Levelling and equipotential surfaces

Regardless of where it is, the level is always perpendicular to the vertical of the equipotential surface where they meet. Thus, the levelling route between points A and B (Figure 34) starts on the equipotential surface located above point A. The difference in elevation measured between points A and 1 (δn_1) is less than the orthometric height difference along the vertical at point B (δH_1). For the route from A to B, the sum of the differences in elevation (Δn_{AB}) is less than the sum of the orthometric height differences (ΔH_{AB}). For the route between points D and B, the phenomenon is the opposite. The difference in elevation measured between points D and 2 (δn_2) is greater than the orthometric height difference along the vertical at point B (δH_2), therefore $\Delta n_{DB} > \Delta H_{DB}$. The result is that levelling from A to D via B would not give the correct orthometric height at point B. Furthermore, there would be a closing error at point D. To make the differences in elevation independent from the route followed, it is necessary to introduce orthometric corrections based on gravity. For the example in Figure 34, the orthometric height difference for point B is:

$$\Delta H_B = \Delta n_{AB} + \Delta v_{AB} = \Delta n_{DB} + \Delta v_{DB} \quad (14)$$

Where Δv is the correction to make to the difference in elevation measured between two points. This correction can take different forms, depending on the recommended approach. For example, Heiskanen and Moritz (1967) propose the following correction:

$$\Delta v_{AB} = \sum_A^B \frac{g_i - \gamma_0}{\gamma_0} \delta n_i + \frac{\bar{g}_A - \gamma_0}{\gamma_0} H_A - \frac{\bar{g}_B - \gamma_0}{\gamma_0} H_B \quad (15)$$

Where g_i is the gravity measured with the difference in elevation of δn_i, $\bar{g}_A$ and $\bar{g}_B$ are the average gravities at points A and B and γ_0 is an arbitrary constant. The problem with this correction is that it is necessary to approximate several of these variables. For instance, the average gravity represents the average between the gravity at point A and the gravity on the geoid along the vertical passing through point A. But it is impossible to measure the gravity on the geoid since it is measured on the terrain. However, equations exist that can estimate the average value from a measured value.

According to equation (15), the gravity must be measured at every difference in elevation. Bomford (1971) and Levallois (1970) propose measuring the gravity according to a certain interval based on the topography. Measurements are spaced farther apart on flat terrain than on mountainous terrain. There are other forms of orthometric correction. One of them is presented in 6.3.2.

A priori, spirit levelling is a technique that seems relatively simple. In the light of this section, obtaining orthometric height differences is much more complex. In this guide, the expression "geodetic levelling" is used to designate the more rigorous method that includes gravity (measured or normal) in the spirit levelling. This term has been in use for a long time to distinguish this method from simple spirit levelling.

6.3 Height Reference System

The height reference system is sometimes called the "height datum," "vertical datum" or "mean sea level." Implementing such a system generally relies on two phases: definition (the system) and realization (the frame).

Definition consists in choosing the reference surface to which the zero height will be assigned, and the parameters and conventions that should be implemented to determine the height at a given point. Realization consists in giving physical representation to the datum for the area by establishing bench marks that will form the height system, taking measurements and determining the heights of these bench marks.

Two systems are presented in this section. The first system is the mean sea level. The practice of using this reference surface goes back more than a thousand years. The second system is the one used in Canada, which is CGVD28.

6.3.1 Mean Sea Level

Long considered a large surface that only varied with the tides, the sea has been used as a reference surface for calculating heights. To give a physical representation to this surface, the only thing needed is to install a tide gauge somewhere, connect it to a bench mark by levelling and take measurements of movements of the sea over several years to determine the zero value that corresponds to mean sea level (MSL). The tide measurements must be taken long enough to cover all the cycles inherent to the tides. The longest of these cycles lasts about 19 years.

Once the reference surface is known, the bench mark has a known orthometric height. From there, precise levelling is done on all the bench marks that make up the height network. To ensure that the geoid remains the reference surface, orthometric corrections are made to the differences in elevation. There could be several tide gauges in large areas. Precise levelling was sometimes done between these tide gauges.

At one time, it was common to assume that MSL coincided with the geoid in practice. Today, with improved knowledge of the physics of the Earth and its large bodies of water, it is now recognized that MSL does not coincide with the geoid. All the oceans do not form a uniform surface. Elements such as currents, differences in temperature, salinity and density create a topography on this surface in relation to the geoid. This sea surface topography (SST) has bumps and hollows like terrestrial topography, but to a much lesser extent. Thanks to satellite altimetry (Figure 35) (NASA, 2007), it is possible to measure the orthometric height of the ocean's surface (H_{SST}) and thereby show its topography. The height is calculated by:

$$H_{SST} = h_{Sat} - D - N$$

Where h_{Sat} is the geodetic height of the altimeter satellite, D is the distance measured by the altimeter satellite at the water's surface and N is the geodetic height of the geoid.

LeGrand et al. (2003) estimate that this topography varies from approximately -1.8 m to +1.2 m in relation to the geoid. Figure 36 shows the average SST from 1992 to 2003 (Stewart, 2007).

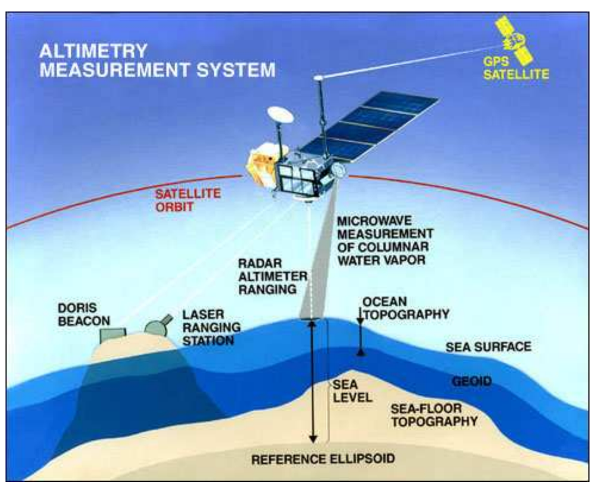


Figure 35: Satellite altimetry

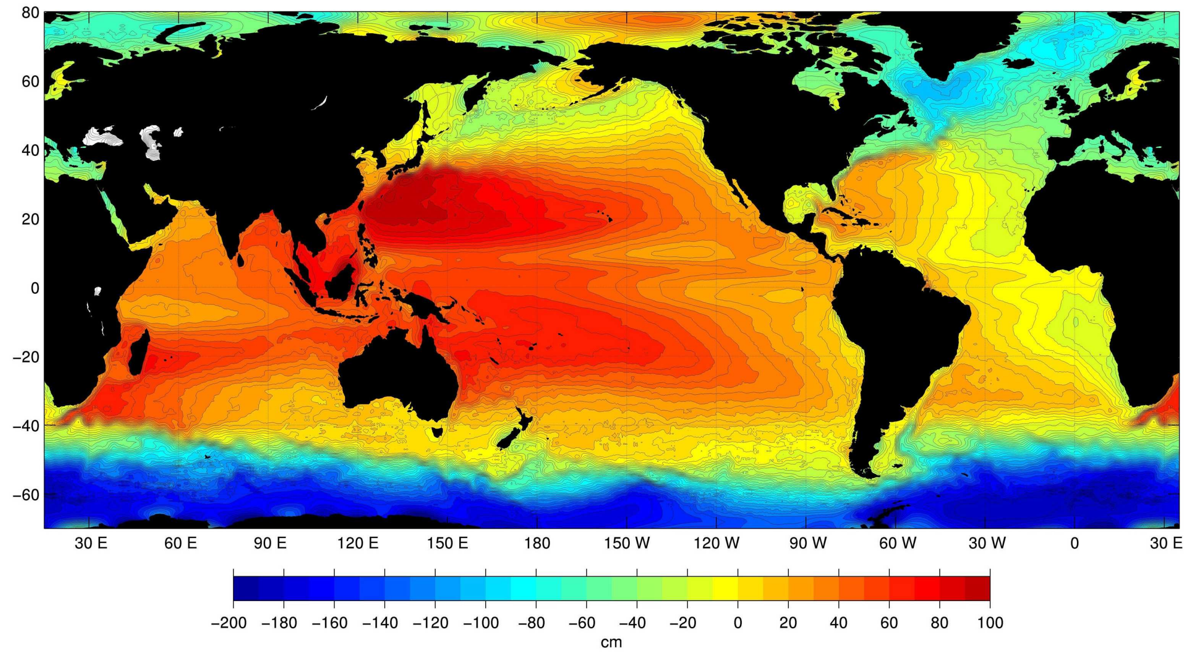


Figure 36: Sea surface topography

Figure 37 summarizes what is measured at a tide gauge. The equipotential surface W_{MSL} represents the mean sea level defined by measurements taken over a long period of time. It is zero height. The difference in height (H_{SST}) between the geoid (W_0) and W_{MSL} is caused by the sea surface topography. H_T is the height at the tide gauge and it is determined once MSL is established. The bench mark height (H_B) is calculated after levelling (ΔH_{B-T}).

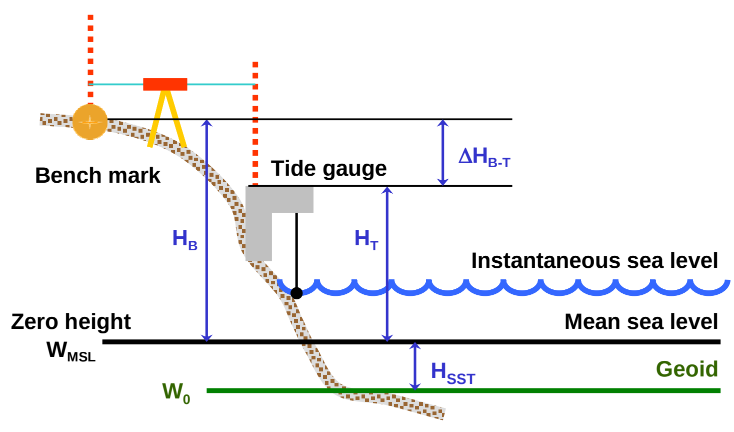


Figure 37: Mean sea level

Other phenomena can influence the location of MSL. For example, if the tide gauge is in a sector where there is uplift or subsidence of the ground caused by post-glacial rebound, among other things, it will observe a relative drop or rise in sea level. A rise in sea level caused by melting ice can also bias the measurements and skew the results. Between two 19-year periods that are sufficiently far apart in time, the MSL, or 0 height, will not be in the same place. It is therefore impossible to fix mean sea level adequately for a very long period. Thus, as Levallois (1969, p. 378) wrote [translation]: “The definition of zero ends up taking on a purely conventional aspect that corresponds to a given epoch” at a given place. The tide gauge alone cannot determine movement of the ground in relation to movement of the sea. Methods to do this are presented in this and the following chapter.

Given that SST is variable, tide gauges installed along a very long shoreline will all have a different W_{MSL} and H_{SST}. Geodetic levelling between these tide gauges could measure a difference in height between them. In reality, it is the difference in height between the H_{SST} of each site that is measured. In such a situation, establishing a reference surface depends on the solution chosen.

One solution would be to adjust the differences in elevation by using only one tide gauge as a reference point (T_1 in Figure 38). The surface would be an equipotential surface if the orthometric corrections are applied properly. The other tide gauges would then have a height difference in relation to the mean sea level that they observed. This approach was not very logical at one time since mean sea level coincided with the geoid.

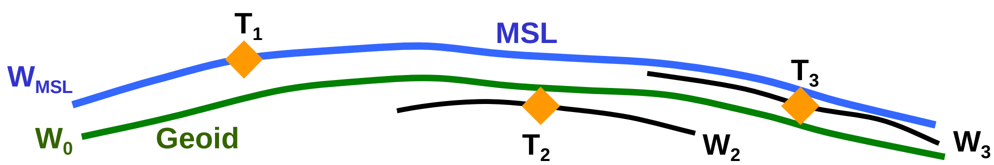


Figure 38: Mean sea level and one tide gauge

Another solution would be to minimize these height differences at the tide gauges by adding a geometric constant to the height of the reference tide gauge (T_1 in Figure 39). By displacing the entire W_{MSL} surface established at the reference tide gauge in this way, the discrepancy between this new surface and the other W_{MSL} surfaces would be minimal. However, this new surface would no longer be an equipotential surface, since equipotential surfaces are not parallel.

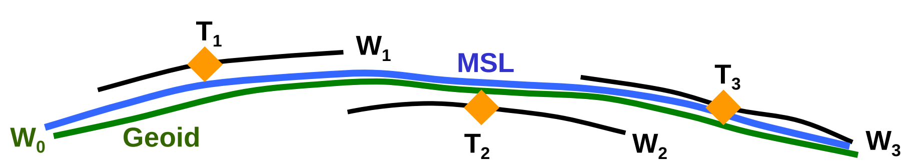


Figure 39: Mean sea level and one tide gauge plus a constant

A third solution would be to constrain the differences in elevation at each W_{MSL} surface measured (Figure 40). The new surface defined by the levelling is no longer an equipotential surface. In addition, it undulates on either side of the geoid. Today, this approach is not to be used, but it was the most logical at the epoch when mean sea level coincided with the geoid.

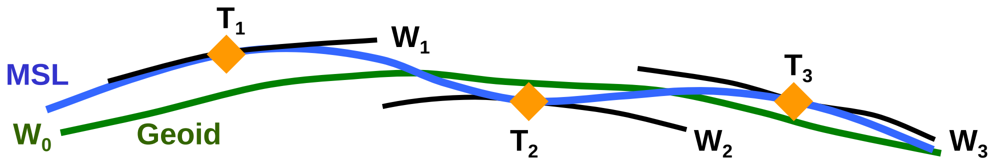


Figure 40: Mean sea level and three tide gauges

To summarize, mean sea level can be considered as a reference surface, but at one location and for one epoch only.

6.3.2 CGVD28

The vertical datum system used in Canada is CGVD28 (Canadian Geodetic Vertical Datum of 1928). By definition, mean sea level is the reference surface and the heights are orthometric. The realization of CGVD28 is based on the 1928 adjustment done by the Geodetic Survey of Canada with 37,000 km of levelling done from 1906 to 1928 (Cannon, 1929; GSC, 1935). Adjustment was constrained to six points: mean sea level observed at five tide gauges (three for the Atlantic and two for the Pacific) and one bench mark at Rouses Point (New York, USA). The tide gauges were located at Halifax and Yarmouth, Nova Scotia, at Pointe-au-Père, Quebec, and at Vancouver and Prince Rupert, British Columbia. The height of the bench mark at Rouses Point came from Canada–US work, where the final value was determined at a conference in March 1925 (Cannon, 1935). The red area in Figure 41 (Cannon, 1929) shows the Canadian territory covered by the adjustment of 1928.

Figure 41: Levelling of 1928

Corrections were made to the differences in elevation (Cannon, 1929). The calculations for these corrections came from work by Bowie and Avers (1914). They based their work on normal gravity (γ) using the following equation:

$$\gamma = \gamma_{45}(1 - \alpha\cos 2\phi + \beta\cos^2 2\phi - k\mathrm{H}) \tag{16}$$

Where α and β are constants, H is the height, ϕ is the latitude, k is a variable that is a function of H and ϕ, and γ_{45} is normal gravity at mean sea level at latitude 45° (9.80624 m/s^2).

By using equation (16) in equation (11) and integrating the latter, the following “orthometric” correction is the result:

$$\mathrm{dH} = -2\alpha\sin 2\phi\left[1 + \left(\alpha - \frac{2\beta}{\alpha}\right)\cos 2\phi\right]\mathrm{H}\mathrm{d}\phi \tag{17}$$

Where α = 0.002644 and β = 0.000007. H represents the mean height, ϕ is the mean latitude and dϕ is the difference in latitude between the starting point and the arrival point of the difference in elevation.

In theory, equation (11) is integrated according to the real gravity (g). By integrating it using normal gravity (γ), “normal” corrections are the result. With equation (17), no correction is applied if the levelling is done along a parallel (dϕ = 0). This is the peculiarity of normal gravity. In the definition for CGVD28, the reference surface is the geoid and the heights are orthometric. By applying “normal” corrections to the heights, they are no longer pure orthometric heights. Corrected this way, they are called normal orthometric heights.

The 1928 adjustment established the Canadian height reference surface. All levelling done thereafter to densify the network was based on the heights from this adjustment, despite the events below.

In 1929, the US adjusted more than 106,000 km of levelling (including the Canadian data). The adjustments were constrained to the mean sea level observed by 26 tide gauges, including the five used to establish CGVD28. The Sea Level Datum of 1929 (SLD29) became the height reference surface for the US. For the Canadian points, the heights in this adjustment do not correspond to the heights obtained in the Canadian adjustment of 1928. Canada kept its solution of 1928.

Cannon (1935) noted that the work done in 1929 and 1930 by the US had changed the height of the bench mark at Rouses Point. Cannon adjusted the 1928 data again, keeping the Rouses Point bench mark free. The height of the bench mark was 3 cm lower than the 1925 value.

Given that the solution of 1928 was being used more and more, it was agreed to not broadcast the results of the new adjustment.

In 1935, the results of the adjustment of 1928 officially became the height reference system for Canada subsequent to approval of order 630 on March 11, 1935 (Appendix C). By this order, the height reference system was officially named the Canadian Geodetic Datum (CGD).

Upon reading the order, it is clear that the CGD is a height reference system. Outside its context, it leads to confusion. First, the term “Geodetic Datum” is generally associated with a conventional reference system such as NAD27. In the 1970s, GSD therefore renamed it the Canadian Geodetic Vertical Datum of 1928. Adding the year of adjustment made it possible to differentiate it from the US SLD29. The CGD, or CGVD28, was sometimes wrongly associated with SLD29, commonly called “Mean Sea Level of 1929” or MSL29.

Second, because of the name given in the order, for many decades users associated the heights in this system with geodetic heights. But the geodetic height is the curvilinear coordinate representing the distance between a point in space and the surface of an ellipsoid (section 3), and not in relation to the geoid. The word “geodetic” in the name indicates that geodetic methods were used to correct the levelling measurements (geodetic levelling). Special attention must be given when a document mentions geodetic heights. In the large majority of cases, the document refers to orthometric heights.

On May 10, 1973, the US changed the name of SLD29 to the National Geodetic Vertical Datum of 1929 (NGVD29) (NGS, 2008). The NGVD29 was replaced in the early 1990s by the North American Vertical Datum of 1988 (NAVD 88) (Zilkoski et al., 1992). In NAVD88, the orthometric corrections are based on measurements of gravity and the network’s adjustment was constrained to a single point: the tide gauge at Rimouski. Although Canada participated in designing NAVD88, the country has not adopted it yet as a new height reference system.

6.3.2.1 Main Errors in CGVD28

When CGVD28 was created, unknown phenomena or approximations in the calculations introduced errors and biases. The two main sources that produced these errors and biases are described in the paragraphs below. Their consequences are illustrated in Figure 42 adapted from GSD (2009a).

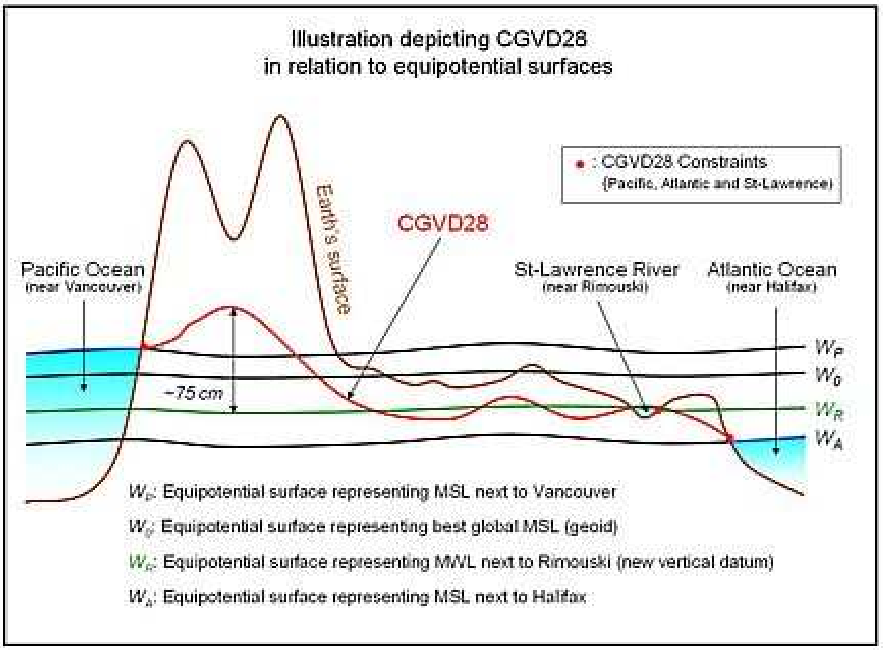


Figure 42: CGVD28 and equipotential surfaces

At the epoch when the datum was designed, it was acknowledged that mean sea level should in practice coincide with the geoid. The Atlantic and Pacific oceans thus rested on the same equipotential surface. Today, because of SST, this assumption is no longer true (section 6.3.1). Recent analyses show that the level of the Pacific Ocean is about 50 cm higher than the level of the Atlantic Ocean. The creation of CGVD28 resembles the solution presented in Figure 40. By constraining the surface to pass through five different equipotential surfaces, CGVD28 was not able to meet one of the parameters of its definition, that of being an equipotential surface. Figure 42 presents a simplified version of the distortion affecting CGVD28. The five constraint points created a downslope from west to east in the surface.

For validation purposes, Cannon (1935) adjusted the 1928 data by constraining the calculations to the Halifax tide gauge only. He observed that the heights obtained for the Vancouver and Prince Rupert tide gauges were higher by 0.26 m and 0.51 m, respectively. Unaware of the existence of SST, he attributed this difference to the predominance of the winds that made the ocean swell along the coast.

During levelling, the differences in elevation measured in the field must be corrected to obtain the differences in orthometric height. According to equation (15), gravity measurements must be taken regularly. However, the “orthometric” correction, as formulated in equation (17), is based on normal (theoretical) gravity and varies mainly according to the difference in latitudes. This introduces a bias in the correction. Using measured gravity would change the height of the Rockies by several decimetres.

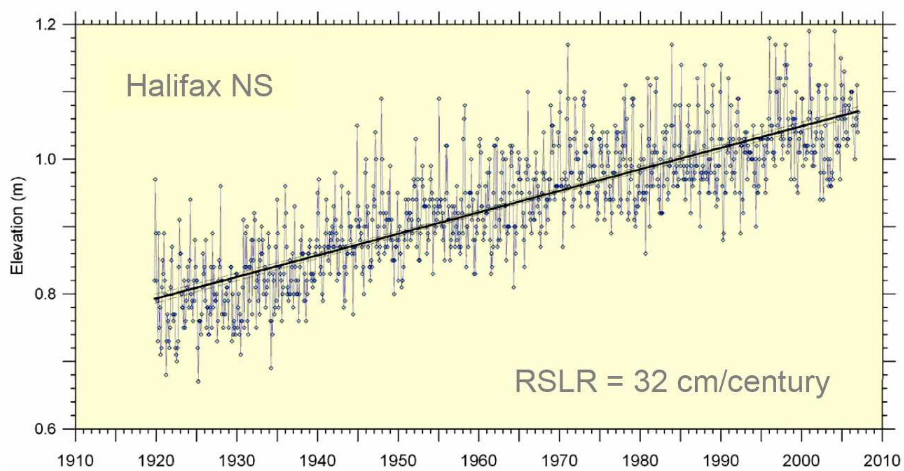
6.3.2.2 Limitations of CGVD28

The main limitation of CGVD28 is its access in the field. Using geodetic levelling to determine the geoid limits access to the areas covered by this levelling only. In this case, only narrow portions of the datum are accessible. To summarize, if there is no geodetic levelling, there is no CGVD28.

Starting in 1928, work to densify and validate the Canadian levelling network was carried out. The initial frame was used as a control for this work. For instance, one levelling done in 1968 is based on the heights established in 1928. If this levelling is in a sector that is strongly influenced by post-glacial rebound, then today, in absolute terms, the heights are biased. For example, if the rebound velocity is 4 mm/yr, then the control points of 1928 are 0.16 m too low in 1968. In 2008, the heights in this sector would be biased by 0.32 m. However, in relative terms, there would be no difference since the entire sector is lifting up at the same velocity.

In 1928, inhabited regions in Canada were not connected to CGVD28 (Figure 41) because of their isolation. For instance, Prince Edward Island, Vancouver Island and, later, Newfoundland, did not have a network from this adjustment. Access to CGVD28 was done by deploying tide gauges (Jones, 1973). This approach complied with the definition of CGVD28 since it defined mean sea level at each location. Knowing that mean sea level varies from place to place, and also from epoch to epoch, it is very possible for a slight difference to exist between the surface obtained for each tide gauge and the surface of CGVD28 that is based on five other tide gauges. Today, in the light of current knowledge, each of the sites given in the example would be considered to be local height reference systems.

Redoing the datum in 2008 with the same approach used in 1928 would put the reference surface at a different level. For example, the Halifax region has to deal with two phenomena: subsidence caused by post-glacial rebound and a rise in sea level. Figure 43 (Forbes, 2008) shows the relative rise in sea level measured at Halifax, and he estimates it at 3.2 mm/yr. With subsidence estimated at 1.7 mm/yr (Forbes, 2008), the real rise in sea level is 1.5 mm/yr. Thus, in 2008, the mean sea level, or zero height, would be established about 0.26 m higher than the zero height of 1928 if the two phenomena are disregarded. In reality, the sea level has risen only 0.12 m since 1928. CGVD28 is a static system that does not take the dynamics of the Earth into account.



RSLR: Relative sea-level rise

Figure 43: Relative rise in sea level at Halifax

6.3.3 Altimetry Infrastructure

Today, CGVD28 is accessible across Canada through more than 80,000 first-order bench marks spread over approximately 150,000 km of levelling lines. Most of them are located in southern Canada. Figure 44 (Véronneau et al., 2006) shows the current state in the area of Canada that has access to CGVD28 through the first-order network. In Quebec, this network was densified by government (departments or government corporations) or municipal organizations. Table 28 shows the hierarchy of the altimetry infrastructure as it was established in Quebec.

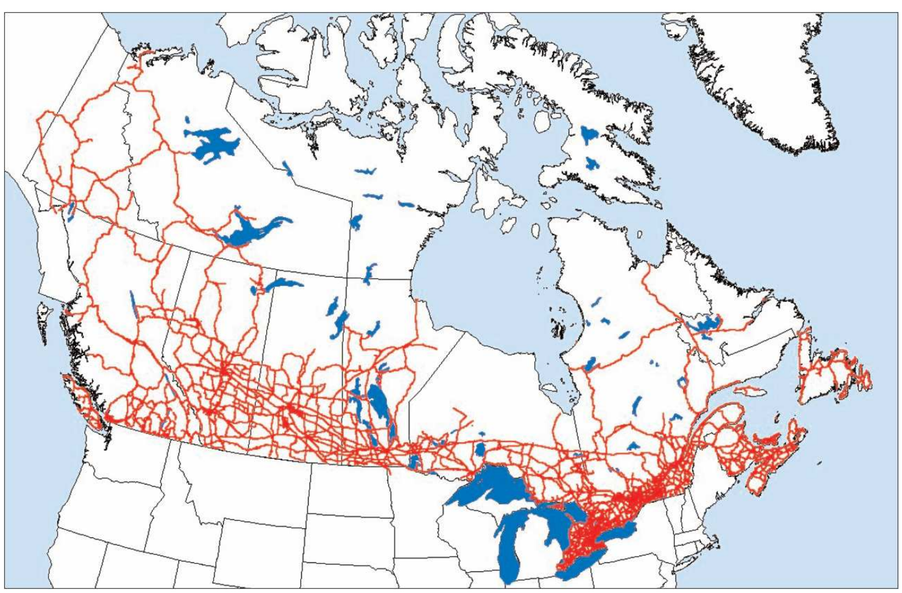


Figure 44: The first-order network in Canada

Table 28: Altimetry infrastructure in Quebec

	Order										
Network	1			2			3			4	
Environment	A	B	C	A	B	C	A	B	C	A	B
Minimum distance between levelling lines (km)	10-20	30-50	200-400	4-8	10-20	50-100	1-2	3-6	15-30	0.3-0.6	0.5-2.0

Network
1: Primary
2: Secondary
3: Tertiary
4: Complementary

Environment
A: Urban
B: Semi-urban
C: Not developed

A metric quality criterion is associated with the height of each bench mark. The DRG uses four categories, or classes, to describe the metric quality of the height. Heights that do not meet the metric criteria are not classified (class D) and are not available for users. The classification is done based on the standard deviation (σ) of the height resulting from the data adjustment. Table 29 shows the maximum permitted limit for a given class and order.

Table 29: Classification criteria (m) in the altimetry infrastructure

		Order										
		1A	**1B**	**1C**	**2A**	**2B**	**2C**	**3A**	**3B**	**3C**	**4A**	**4B**
Class	1	0.014	0.020	0.040	0.014	0.020	0.026	0.012	0.018	0.050	0.020	0.030
	2	0.020	0.028	0.057	0.020	0.028	0.037	0.017	0.025	0.071	0.028	0.042
	3	0.028	0.040	0.080	0.028	0.040	0.052	0.024	0.036	0.100	0.040	0.060

6.4 Geoid Model

We stated above that the geoid (W_0) is the equipotential surface that best represents, in a least square sense, the oceans at rest. This surface is directly related to the gravitational field. Because it is intangible, the geoid cannot be measured directly. However, it can be modelled based on gravity measurements. These measurements can come from various sources: terrestrial, airborne, or satellite. The recent presence of satellite measurements in geoid modelling has improved the accuracy of the models.

The spatial representation of the geoid, or its modelling, is done with a terrestrial reference system. The result of this representation is a relatively dense grid where the geodetic height of the geoid is determined at each node. The nodes are also known in position (latitude and longitude). The grid density and its geographic coverage are established according to its use. The set of data forms a model.

Since the geoid is a specific surface, its geodetic height is called the geoid undulation (N). This name comes from the fact that the surface undulates on either side of the ellipsoid (Figure 45). The relation between geodetic height (h), orthometric height (H) and geoid undulation (N) is expressed as:

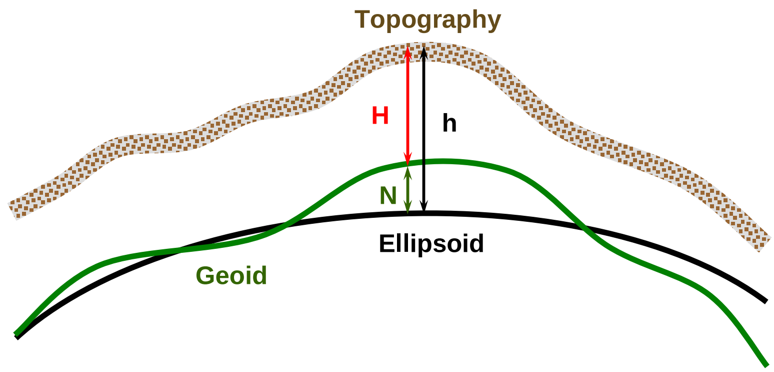


Figure 45: Geoid undulation

$$h = H + N \tag{18}$$

Geoid undulation forms the link between the mathematical world (geodetic height) and the gravimetric world (orthometric height).

The value of N for a single point on a geoid can vary depending on the shape, orientation or location of the ellipsoid. Figure 46 presents the situation with two ellipsoids. The position of the ellipsoids is exaggerated to clearly illustrate the phenomenon. Regardless of the location of the ellipsoid, the geoid is always in the same place because it is a real surface, whereas the ellipsoid is an arbitrary theoretical surface. Thus, a point on the ground has the same orthometric height regardless of the different values of N and h since $h_A - N_A = h_B - N_B$.

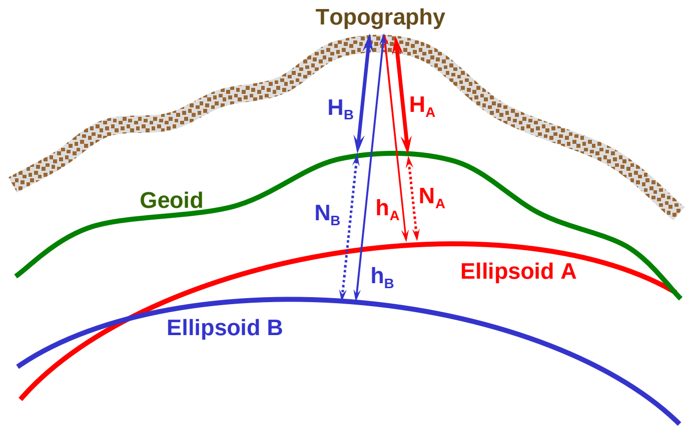


Figure 46: Geoid and ellipsoids

Several geoid models exist. Some are global and others are regional. The best known global model is EGM96 (Earth Gravitational Model of 1996) (Figure 47) (NASA, 2009). Its main characteristics are presented in Table 30. Its 15' × 15' grid represents a surface of 27.8 km (north) by 19.5 km (east) in the Quebec City region.

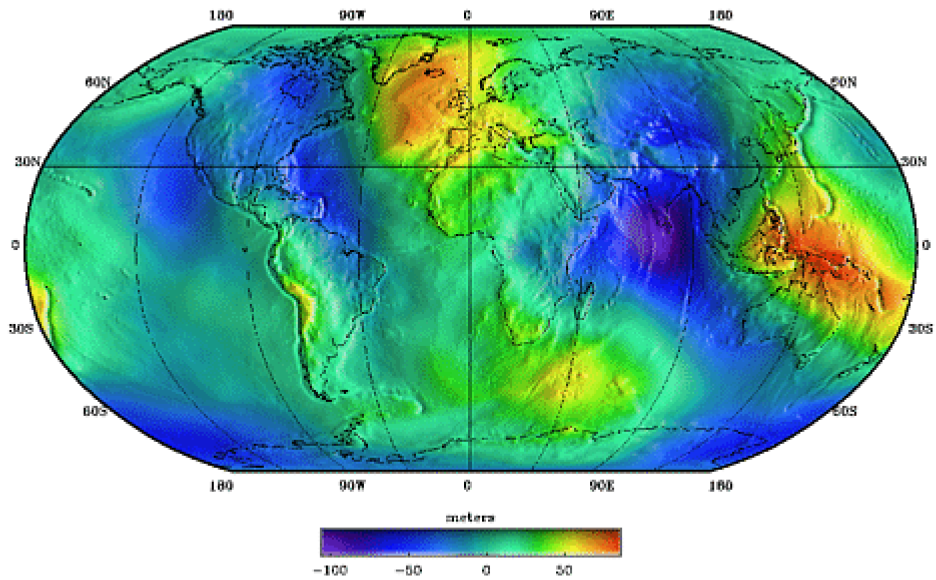


Figure 47: EGM96 global geoid

Regional models are calculated on a relatively large portion of the Earth. Because they have limited sections to model, the grid is generally denser and there are more gravity measurements in the calculations. Other sources of data can also improve the quality of the model. For specific needs, the model can use an equipotential surface (W) that is different from the geoid (W_0). For example, when establishing a height reference system, the choice of the equipotential surface to use as the "zero" reference is essential. This surface can be an equipotential surface other than the geoid.

GSD has developed several models for Canada over the last two decades. The most recent are CGG2000 and CGG2005 (Figure 48) (Véronneau, 2006) where CGG stands for Canadian Gravimetric Geoid. Table 30 presents the main characteristics of these two models. Their 2' × 2' grid represents a surface of 3.7 km (north) by 2.6 km (east) in the Quebec City region.

Table 30: EGM96, CGG2000 and CGG2005 characteristics

	EGM96	**CGG2000**	**CGG2005**
Reference Frame	WGS84 (G873)	ITRF97	ITRF2000
Ellipsoid	WGS84	GRS80	GRS80
W_0	62,636,856.88 m^2/s^2	62,636,855.8 m^2/s^2	62,636,856.88 m^2/s^2
Grid	15' × 15'	2' × 2'	2' × 2'

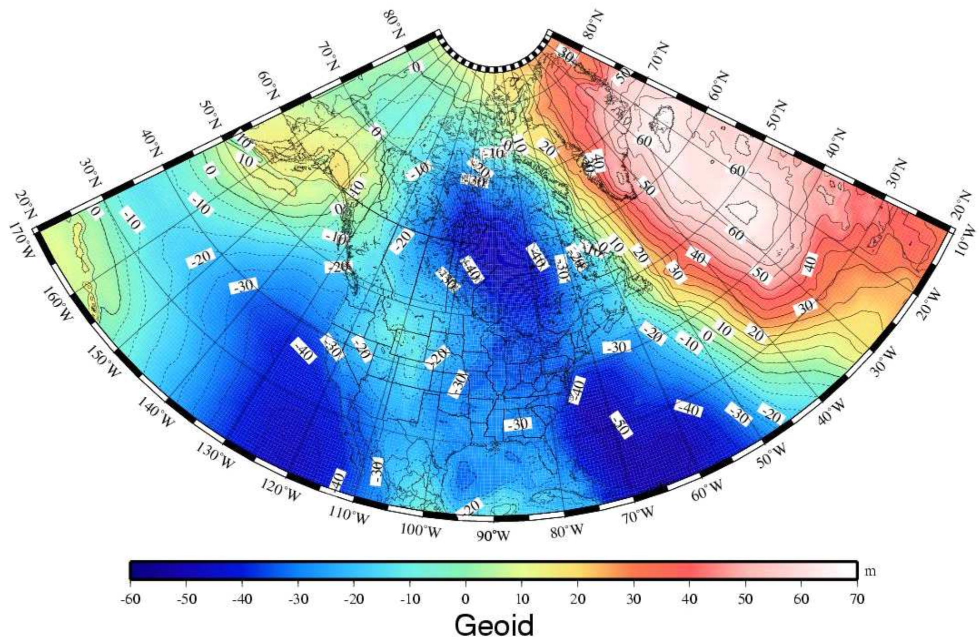


Figure 48: Canadian CGG2005 geoid

Although the Canadian models use practically identical reference frames, the value of N for the same point differs by about 0.1 m. These models do not use the same equipotential surface to represent the geoid (W_0). However, the difference in undulation (ΔN) between two points is relatively the same for the two models. This slight difference comes from the improvement in the quality and quantity of data used to calculate the recent model, among other things. For instance, CGG2005 was able to use the gravity data from the GRACE (Gravity Recovery and Climate Experiment) satellite mission. Numerical examples are presented below.

Since it is directly tied to the Earth, the form of the geoid changes constantly, but very slightly, as the masses move. For example, post-glacial rebound is lifting W_0 very slowly. Thus, two geoid models created at different epochs could have slightly different values. According to GSD (GSD, 2009b), the geoid is lifting about 1.5 mm/yr at the location of the maximum post-glacial rebound uplift.

6.4.1 Hybrid Geoid

The arrival of space-based positioning, such as GPS in the 1980s, highlighted the weaknesses, and especially the incompatibility, between CGVD28 and modern geodetic datums. The NAD83 (CSRS) terrestrial reference frame used in Canada makes it possible to obtain geodetic heights with a very high degree of accuracy. To convert these geodetic heights to CGVD28 orthometric heights, using equation (18) with a geoid model such as CGG2000 initially appears to be a simple solution. In reality, NAD83 (CSRS), CGVD28, and the CGG2000 geoid model are incompatible with each other.

To understand these incompatibilities as illustrated in Figure 49, it is necessary to review certain concepts presented earlier. Firstly, CGVD28, as it was designed, is not an equipotential surface (Figure 42). Regardless of the value assigned to W_0 to define the geoid, this vertical frame cannot be represented by a geoid model.

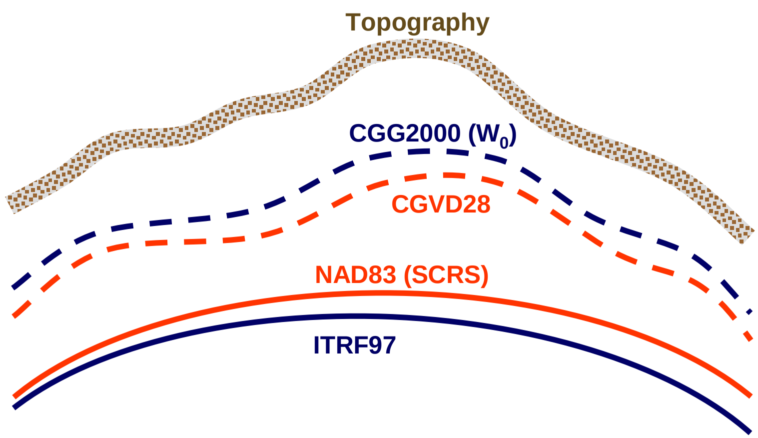


Figure 49: NAD83 (CSRS), CGVD28 and CGG2000

Secondly, the undulation values of the CGG2000 model are calculated in ITRF97, whereas the geodetic heights are calculated in NAD83 (CSRS). However, it is possible to transform the undulation values from the CGG2000 model to NAD83 (CSRS) since the transformation parameters between ITRF97 and NAD83 (CSRS) are known (section 5.2.1.3). Nevertheless, this transformation does not change the location and shape of the geoid.

Subsequent to these observations, it is clear that equation (18) cannot be applied directly since H_{CGVD28} is not equal to $H_{CGG2000}$. The conclusion therefore is that the geoid undulation of CGG2000 does not meet the need to directly transform the geodetic heights resulting from NAD83 (CSRS) into CGVD28 orthometric heights. This conclusion applies to all gravimetric geoid models.

To solve this problem, GSD "modelled" the surface of CGVD28. To do this, it used GPS to determine the geodetic height of 2,834 bench marks located across Canada in version 3.1 of NAD83 (CSRS). This set of bench marks forms the Supernet (v3.1) and two thirds have an orthometric height. Figure 50 (Véronneau et al., 2001) shows in red the 1,926 bench marks, or control points, retained for calculating the model.

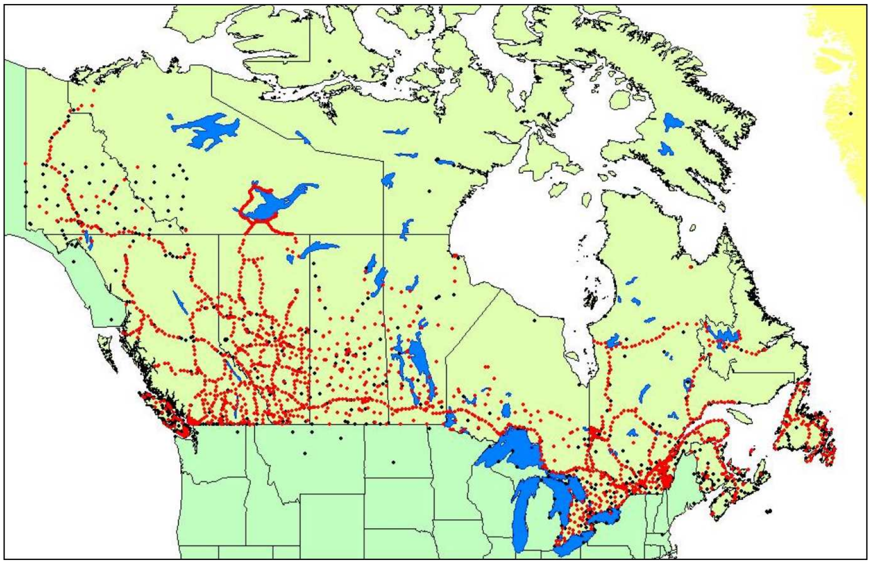


Figure 50: Supernet

The orthometric discrepancy (ε_H) was determined for each bench mark retained using the following equation:

$$\varepsilon_H = (h_{NAD83\ (CSRS)} - N_{CGG2000\ (NAD83)}) - H_{CGVD28} \qquad (19)$$

Where:

$h_{NAD83\ (CSRS)}$: geodetic height in NAD83 (CSRS)

$N_{CGG2000\ (NAD83)}$: geoid undulation of CGG2000 transformed to NAD83 (CSRS)

H_{CGVD28}: orthometric height in CGVD28

From these 1,926 discrepancies, a correction model was created in the form of a grid where each node represents a correction value. By combining this correction model with the CGG2000 geoid model that had already been transformed to NAD83 (CSRS), GSD produced a so-called "hybrid" geoid model with a 2' × 2' grid. This hybrid geoid, called HT2.0 (Height Transformation, version 2.0), makes it possible to obtain a CGVD28 orthometric height directly from a NAD83 (CSRS) geodetic height. Developing hybrid geoid models is an international tendency. Many countries use this type of model to obtain orthometric heights, compatible with their height reference system, from geodetic heights.

The HT2.0 hybrid geoid is particularly efficient in areas around any of the 1,926 bench marks that were used to create it. By applying it to these bench marks, 95% of them have a residual error of less than 5 cm. These points are located mainly in the southern regions of Canada. For the remaining 5%, the residual error can reach several decimetres (Véronneau et al., 2001). Outside the areas covered by the Supernet, HT2.0 must be used with a good knowledge of the region. This warning also applies to regions that do not have bench marks attached to the

CGVD28 infrastructure. The orthometric heights in these areas generally result from local height systems that are more or less compatible with CGVD28. A case is presented in the next section.

6.5 Calculating CGVD28 Heights

The purpose of this section is to present a few numerical examples to show how to use a geoid model to obtain CGVD28 heights from geodetic heights. These examples can also be used to understand the concepts discussed earlier in this chapter. The data analyzed come from the GPS-H software. A brief description of the software is necessary in order to properly understand the examples that follow.

GSD developed GPS-H (Figure 51) to extract the undulation (N) and components (ξ, η) of the deflection from the vertical at a given point from a geoid model, and calculate the orthometric height if the geodetic height is known. It also allows users to transform these values to NAD83 (CSRS) if the original frame of the geoid model is the ITRF. GPS-H uses the term “ellipsoidal height” to designate geodetic height.

GPS-H allows users to transform geodetic heights to orthometric heights using three approaches. The first approach is direct transformation based on equation (18), or $H = h – N$. From the position (latitude and longitude) of a point, GPS-H extracts the geoid undulation (N) from the model and subtracts it from the geodetic height (h).

The second approach consists in adding a bias to the geoid undulation. This bias ($h – N – H$) is calculated with a control point for which the heights H and h are known and N is extracted from the geoid model. If there are several control points, then an average bias is used. If the orthometric heights of the control points are in CGVD28, then the geodetic heights of the other points are transformed to orthometric heights that are compatible with CGVD28 using this simple relation: $H = h – (N + bias)$. Otherwise, they will be compatible with the height system specific to the control points.

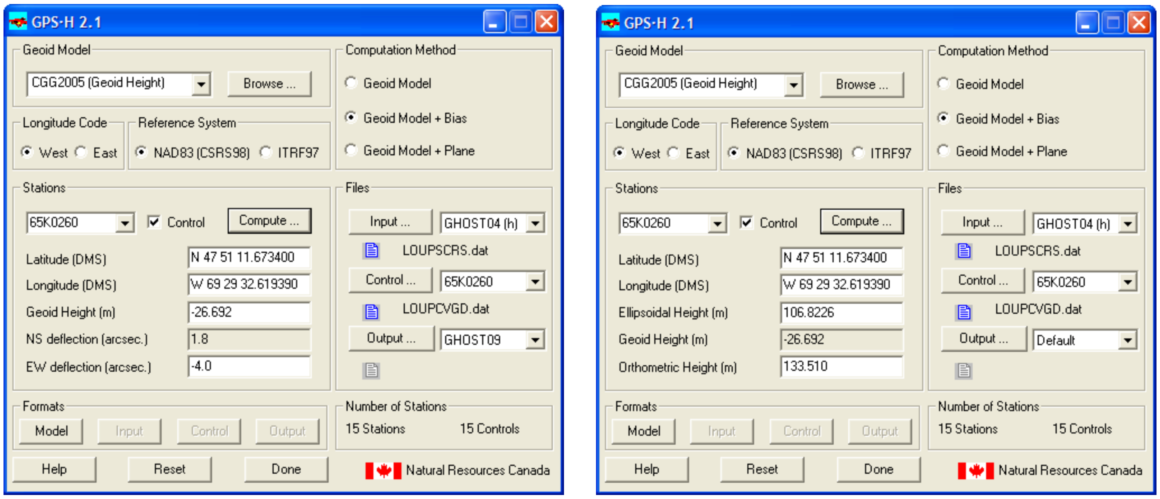


Figure 51: GPS-H software

The third approach consists in adding a correction to the geoid undulation. Using a minimum of five control points, a correction plane is calculated and a different correction is applied to each point by the following equation: H = h – (N + correction). Figure 42 shows that there is a west to east slope in the surface of CGVD28. There could be a small local slope in part of this surface. This approach makes it possible to model the local slope as long as the control points are distributed evenly across the working area. According to the application's help file, this approach is recommended for large areas (50 to 100 km).

6.5.1 Metric Analyses

The first metric analysis presents the effects caused by geoid undulations calculated in two different geodetic reference frames and the application of the HT2.0 hybrid geoid. Table 31 shows the geodetic (h) and orthometric (H) heights for four bench marks, named by their unique number, in the Rivière-du-Loup region. The geodetic heights are in version 2 of NAD83 (CSRS) and the orthometric heights are in CGVD28. Column "N_{83-28}" shows the discrepancy (h – H) between these two frames. In a perfect world, this discrepancy would represent the geoid undulation.

Table 31: Heights (h, H) and height differences for four bench marks

Unique Number	h	H	N_{83-28}
79L073	249.783 m	275.754 m	-25.971 m
79L089	-22.164 m	4.616 m	-26.780 m
83L118	14.776 m	41.401 m	-26.625 m
99K0072	-22.268 m	4.424 m	-26.692 m

In Table 32, column “ITRF” lists the geoid undulations extracted directly from the CGG2000 and CGG2005 models, i.e., in their original frame. Column “NAD83 (CSRS)” gives the geoid undulations extracted from the CGG2000 and CGG2005 models, but transformed to this frame. The difference of about 1 m between the two frames for the same geoid model is due to the fact that the NAD83 system is not geocentric compared with the ITRS. The difference of about 0.1 m between the two geoid models can be explained by the fact that they do not use the same equipotential surface to represent the geoid (W_0) (Table 30).

Table 32: CGG2000 and CGG2005: ITRF and NAD83 (CSRS)

Unique Number	ITRF		NAD83 (CSRS)	
	CGG2000	CGG2005	CGG2000	CGG2005
79L073	-26.602 m	-26.746 m	-25.541 m	-25.685 m
79L089	-27.414 m	-27.543 m	-26.359 m	-26.488 m
83L118	-27.298 m	-27.412 m	-26.249 m	-26.363 m
99K0072	-27.346 m	-27.463 m	-26.295 m	-26.412 m

Table 33 shows the difference between the values in Table 32 and the $N_{83\text{-}28}$ values in Table 31 for each point. These values represent the discrepancy that users would obtain if they transformed their NAD83 (CSRS) geodetic heights to GCVD28 orthometric heights directly with a gravimetric geoid model.

Table 33: CGG2000 and CGG2005: Difference with $N_{83\text{-}28}$

Unique Number	ITRF		NAD83 (CSRS)	
	CGG2000	CGG2005	CGG2000	CGG2005
79L073	-0.631 m	-0.775 m	0.430 m	0.286 m
79L089	-0.634 m	-0.763 m	0.421 m	0.292 m
83L118	-0.673 m	-0.787 m	0.376 m	0.262 m
99K0072	-0.654 m	-0.771 m	0.397 m	0.280 m

Table 34 presents the geoid undulation (N_2), orthometric height (H_2) obtained by direct transformation with the HT2.0 model, and the difference (ΔH) between this height and the known CGVD28 value (Table 31, column “H”). The small difference shows a high degree of consistency of the HT2.0 model when it is used in a sector that is properly controlled by several Supernet (Figure 50) bench marks. Let us recall that the difference (ΔH) includes the errors contained in the geodetic height (h), orthometric height (H) and the HT2.0 hybrid geoid undulation (N_2), as well as the height discrepancy between versions 2 and 3.1 of NAD83 (CSRS).

Table 34: HT2.0 – Rivière-du-Loup region

Unique Number	N_2	H_2	ΔH
79L073	-25.981 m	275.764 m	0.010 m
79L089	-26.792 m	4.628 m	0.012 m
83L118	-26.622 m	41.398 m	-0.003 m
99K0072	-26.683 m	4.415 m	-0.009 m

The second analysis is in a sector that has no control points from the Supernet. This analysis highlights the weakness of the HT2.0 hybrid geoid model in such areas and proposes methods to transform the geodetic heights using the GPS-H software. Table 35 shows the geodetic (h) and orthometric (H) heights for eight bench marks, named by their unique number, in the Kuujjuarapik region. The geodetic heights are in version 2 of NAD83 (CSRS) and the orthometric heights are initially in CGVD28. Column "N_{83-28}" shows the discrepancy (h – H) between these two frames. In a perfect world, this discrepancy would represent the geoid undulation.

Table 35: Heights h and H, and height differences for eight bench marks

Unique Number	h	H	N_{83-28}
72KA102	-3.118 m	38.621 m	-41.739 m
72KA116	-12.269 m	29.493 m	-41.762 m
86KS001	-33.607 m	8.187 m	-41.794 m
86KS002	-31.813 m	9.992 m	-41.805 m
86KS003	-28.748 m	13.042 m	-41.790 m
86KS004	-4.364 m	37.370 m	-41.734 m
89L310	-8.009 m	33.726 m	-41.735 m
89L314	-40.703 m	1.101 m	-41.804 m

Bench marks 89L310 and 89L314 are part of the Supernet, but they were excluded from the process to produce the HT2.0 model because they were not tied to the CGVD28 infrastructure by geodetic levelling. The orthometric heights in this region must be considered as if they are part of a local height reference system.

Table 36 shows an extreme case of error that can be produced by direct transformation with the HT2.0 model in a local height system. For the region studied, there is a difference (ΔH) of about 1 m between the known height (Table 35, column "H") and the one obtained (H_2) after direct transformation.

Table 36: HT2.0 – Kuujjuarapik region

Unique Number	N_2	H_2	ΔH
72KA102	-42.690 m	39.572 m	0.952 m
72KA116	-42.705 m	30.436 m	0.943 m
86KS001	-42.732 m	9.125 m	0.939 m
86KS002	-42.738 m	10.925 m	0.932 m
86KS003	-42.731 m	13.983 m	0.940 m
86KS004	-42.683 m	38.319 m	0.949 m
89L310	-42.685 m	34.676 m	0.950 m
89L314	-42.749 m	2.046 m	0.946 m

Direct transformation from the HT2.0 model should be avoided in a sector where the heights result from a local system. However, the GPS-H software offers two methods to transform geodetic heights to local heights or CGVD28 heights: transformation by bias and transformation by plane. To present these two methods, five of the eight bench marks are used as control points. Bench marks 72KA102, 86KS003 and 86KS004 are used as validation bench marks to illustrate the accuracy of these two methods. The examples below use real data only to explain the behaviour of these two methods.

Table 37 presents the results obtained with the transformation by bias. Column "N_B" gives the geoid undulation, corrected by the bias, used to calculate the local orthometric heights (H_B). Column "ΔH_B" gives the difference between H_B and the known value (Table 35, column "H"). Undulations N_{2005} come from model CGG2005 transformed to NAD83 (CSRS).

Table 37: Height transformation with a bias

Unique Number	N_{2005}	Bias	N_B	H_B	ΔH_B
Control Point					
72KA116	-42.399 m	0.636 m	-41.763 m	29.494 m	0.001 m
86KS001	-42.425 m	0.636 m	-41.790 m	8.183 m	-0.004 m
86KS002	-42.432 m	0.636 m	-41.796 m	9.983 m	-0.009 m
89L310	-42.379 m	0.636 m	-41.743 m	33.734 m	0.008 m
89L314	-42.443 m	0.636 m	-41.808 m	1.105 m	0.004 m
Validation Bench Mark					
72KA102	-42.384 m	0.636 m	-41.748 m	38.630 m	0.009 m
86KS003	-42.425 m	0.636 m	-41.789 m	13.041 m	-0.001 m
86KS004	-42.377 m	0.636 m	-41.742 m	37.378 m	0.008 m

Table 38 presents the results obtained with the transformation by plane. Column "N_P" gives the geoid undulation, corrected by a plane, used to calculate the local orthometric heights (H_P). Column "ΔH_P" gives the difference between H_P and the known value (Table 35, column "H").

Despite the small size of the sector (less than 3 km), there seems to be a slight slope since the difference ΔH_P at the control bench marks is smaller than the one obtained with the transformation by bias. Recall that ΔH_B and ΔH_P include the inaccuracy of the heights, the geoid model and its transformation to NAD83 (CSRS).

Table 38: Height transformation with a plane

Unique Number	N_{2005}	Correction	N_P	H_P	ΔH_P
Control Point					
72KA116	-42.399 m	0.636 m	-41.763 m	29.494 m	0.001 m
86KS001	-42.425 m	0.629 m	-41.796 m	8.189 m	0.002 m
86KS002	-42.432 m	0.633 m	-41.798 m	9.985 m	-0.007 m
89L310	-42.379 m	0.644 m	-41.735 m	33.726 m	0.000 m
89L314	-42.443 m	0.636 m	-41.807 m	1.104 m	0.003 m
Validation Bench Mark					
72KA102	-42.384 m	0.641 m	-41.743 m	38.625 m	0.004 m
86KS003	-42.425 m	0.637 m	-41.788 m	13.040 m	-0.002 m
86KS004	-42.377 m	0.644 m	-41.734 m	37.370 m	0.000 m

The choice of transformation method by bias or by plane depends on several factors, including the desired accuracy. Transformation by bias is preferred for the initial analysis of the working sector, making it easier to detect whether there is a plane in the sector. For the control points, it is better to use bench marks whose orthometric height comes from geodetic levelling. Trigonometric levelling does not provide the same accuracy for the heights. In the transformation, the gravimetric geoid model must also be transformed to NAD83 (CSRS).

To conclude, transformation by bias or by plane is the preferred method to obtain CGVD28 orthometric heights. This type of transformation takes the small local particularities into account that direct transformation with the HT2.0 hybrid geoid model does not, even in sectors controlled by the Supernet.

7 Future Datums

Today, with increasingly accurate measuring and processing techniques and after two decades of data collection, the ITRS realizations, which are the ITRFs, are reaching a degree of stability in terms of accuracy and precision. There is no need to anticipate a major change over the next few years in the ITRS realization, and even less so in its definition. The regional terrestrial reference systems implemented recently that are based on a version of the ITRF should not undergo any changes over the next few years. However, there are some regional terrestrial reference systems and their frames that can still evolve. For instance, the NAD83 regional terrestrial reference system, as it has been realized, presents a certain degree of incompatibility compared with space-based technologies. This incompatibility comes primarily from the offset of about 2 m that exists between its origin and the origin of the global terrestrial reference systems. NAD83 should give way in the medium term to a geocentric regional terrestrial reference system that is compatible with the space-based technologies.

Contrary to the global terrestrial reference systems and their realizations, the height reference systems are entering an era of change that will transform the existing systems. The main change will come from geoid modelling. The challenges in the early 21st century will be to model the geoid so that it represents its true shape as accurately as possible. With an exact, precise geoid model that is referenced in the ITRS, the scientific community will have a unique global reference surface. By its nature, this surface is independent from the Earth's crust. It will therefore be possible to distinguish, among other things, ground motion (uplift or subsidence) from variations in sea level. Careful monitoring of sea levels will become an international issue over the next few years.

Thanks to advances in technology in gravimetry, in particular satellite gravimetry, it is no longer necessary to conduct geodetic levelling to establish a height reference frame. The existing frames are mostly local frames that did not coincide with the geoid but with some equipotential surface in the best case. The quality of the latest geoid models that include satellite measurements shows that they can be used as a reference surface and therefore become a height reference frame if they are combined with a terrestrial reference frame.

Using a geoid model as a height reference surface is an approach that is being analyzed by several countries to replace existing height reference systems. For example, New Zealand uses this approach to unify all its local height networks. In Canada, the Geodetic Survey Division at Natural Resources Canada has implemented a project to modernize the Canadian height reference system. The reference surface for this new system will be a geoid model realized with the most recent gravimetric data and associated to NAD83 (CSRS). The new height reference system should take over in early 2013 and gradually replace CGVD28.

One of the main benefits of the future Canadian Height Reference System (CHRS) will be to obtain orthometric heights (H_{CHRS}) across all of Canada using this simple equation:

$$H_{CHRS} = h_{CHRS} - N$$

Where h_{CHRS} is the NAD83 (CSRS) geodetic height determined by a GNSS technique and N will come from a new geoid model.

The new CHRS will modify the known orthometric heights across Canada. Table 39 presents the preliminary differences (ΔH) between the CGVD28 heights and the heights of the future CHRS (GSD, 2009c) for 15 Canadian cities. Montreal will thus see its heights decrease by about 0.37 m, but there will be practically no change in the differences in elevation. The west to east slope of about 0.86 m in CGVD28 is shown clearly in Table 39. Recall that this slope is caused primarily by poor knowledge of the sea surface topography when CGVD28 was implemented. The future CHRS will offer Canada a vertical datum that is independent from the Earth's crust and the ocean.

In the US, in 2008, the National Geodetic Survey presented its 2008-2018 10-year action plan (NGS, 2009). The plan foresees giving the US a new height reference system by 2018. Realization of this new system will include implementing a North American geoid model calculated in a new regional terrestrial reference system. The NAVD88 height reference system and the NAD83 terrestrial reference system will be replaced in 2018 with modern reference systems on their territory. By using the geoid as a height reference surface, Canada and the US will finally share a common vertical datum.

Table 39:
Height differences (ΔH)
CGVD28 – Future CHRS

City	ΔH
Vancouver	0.22 m
Banff	0.46 m
Edmonton	-0.03 m
Regina	-0.26 m
Winnipeg	-0.32 m
Thunder Bay	-0.04 m
Toronto	-0.34 m
Montreal	-0.37 m
Rimouski	-0.27 m
Halifax	-0.64 m
Charlottetown	-0.47 m
St. John's	-0.37 m
Whitehorse	0.35 m
Tuktoyaktuk	-0.33 m
Yellowknife	-0.36 m

Bibliography

BOMFORD, G. (1971). *Geodesy*, 3rd ed, Oxford, U.K., Clarendon Press, 731 p.

BOURQUE, P.-A. (2010). *Planète Terre : La structure interne de la Terre*, Department of Geology and Geological Engineering, Laval University, [Online]. [http://www2.ggl.ulaval.ca/personnel/bourque/intro.pt/planete_terre.html] (Consulted in February 2010).

BOWIE, W. and H.G. AVERS (1914). *Fourth General Adjustment of the Precise Level Net in the United States and the Resulting Standard Elevations*, Special Publication No. 18, U.S. Coast & Geodetic Survey, Government Printing Office, Washington, D.C., 328 p.

CANNON, J.B. (1929). *Adjustment of the precise level net of Canada 1928*, Geodetic Survey Publication No. 28, Department of the Interior, Ottawa, F.A. Acland, King's Printer, 41 p.

CANNON, J.B. (1935). *Recent adjustments of the precise level net of Canada*, Geodetic Survey of Canada Publication No. 56, Department of the Interior, Ottawa, J.O. Patenaude, King's Printer, 23 p.

CRAYMER, M. (2006a). "The Evolution of NAD83 in Canada," *Geomatica*, vol. 60, no. 2, p. 151-164.

CRAYMER, M. (2006b). "The Evolution of NAD83 in Canada: Addendum," *Geomatica*, vol. 60, no. 4, p. 433.

CRAYMER, M., R. FERLAND and R. SNAY (2000). "Realization and Unification of NAD83 in Canada and the U.S. via the ITRF," *Towards an Integrated Global Geodetic Observing System (IGGOS)*, IAG Section II Symposium, Munich, October 5-9, 1998, International Association of Geodesy Symposia, vol. 120, Berlin, Springer-Verlag, p. 118-121.

DeMETS, C., R.G. GORDON, D.F. ARGUS and S. STEIN (1994). "Effect of recent revisions to the geomagnetic reversal time scale on estimate of current plate motions," *Geophysical Research Letters*, vol. 21, no. 20, p. 2191-2194.

FORBES, D.L. (2008). "Climate change impacts and adaptation within Atlantic Region's coastal zones," *Climate Change Adaptation Workshop for Atlantic Canada*, St. John, N.B., 8 and 9 May 2008.

GEODETIC SURVEY DIVISION (2006). *Earth's Rotation*, Natural Resources Canada, [Online]. [http://www.geod.nrcan.gc.ca/edu/geod/vlbi/vlbi03_e.php] (Consulted in January 2006).

GEODETIC SURVEY DIVISION (2008a). *Frequently Asked Questions*, Natural Resources Canada, [Online]. [http://www.geod.nrcan.gc.ca/faq_e.php] (Consulted in May 2008).

GEODETIC SURVEY DIVISION (2008b). *Active Control Points (ACP)*, Natural Resources Canada, [Online]. [http://www.geod.nrcan.gc.ca/acp/index_e.php] (Consulted in October 2008).

GEODETIC SURVEY DIVISION (2008c). *Canadian Base Network (CBN)*, Natural Resources Canada, [Online]. [http://www.geod.nrcan.gc.ca/cbn/index_e.php] (Consulted in October 2008).

GEODETIC SURVEY DIVISION (2009a). *Height Reference System Modernization – Reference System*, Natural Resources Canada, [Online]. [http://www.geod.nrcan.gc.ca/hm/ref_system_e.php] (Consulted in October 2009).

GEODETIC SURVEY DIVISION (2009b). *Height Reference System Modernization – Images*, Natural Resources Canada, [Online]. [http://www.geod.nrcan.gc.ca/hm/images_e.php] (Consulted in October 2009).

GEODETIC SURVEY DIVISION (2009c). *Height Reference System Modernization – Height Modernization*, Natural Resources Canada, [Online]. [http://www.geod.nrcan.gc.ca/hm/index_e.php] (Consulted in October 2009).

GEODETIC SURVEY OF CANADA (1935). *Annual report of the Director of the Geodetic Survey of Canada for the fiscal year ending March 31, 1935*, Department of the Interior, Ottawa, J.O. Patenaude, King's Printer, 16 p.

HEISKANEN, W.A. and H. MORITZ (1967). *Physical Geodesy*, San Francisco, W.H. Freeman & Co., 364 p.

HENTON, J.A., M. CRAYMER, M. PIRASZEWSKI and E. LAPELLE (2004). *Crustal Deformation Velocities From Episodic Regional Measurements at Canadian Base Network Sites*, poster presented at the AGU Fall Meeting in San Francisco, December 13-17.

HENTON, J.A., M. CRAYMER, H. DRAGERT, S. MAZZOTTI, R. FERLAND and D.L. FORBES (2006). "Crustal Motion and Deformation Monitoring of the Canadian Landmass," *Geomatica*, vol. 60, no. 2, p. 173-191.

INTERNATIONAL GNSS SERVICE (2009). *IGS Tracking Network*, [Online]. [http://igscb.jpl.nasa.gov/network/netindex.html] (Consulted in April 2008).

INTERNATIONAL EARTH ROTATION AND REFERENCE SYSTEMS SERVICE (2004). *IERS Conventions (2003)*, Dennis D. McCarthy and Gérard Petit (IERS Technical Note 32), Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 127 p.

INTERNATIONAL TERRESTRIAL REFERENCE FRAME (2008a). *ITRF2005: Input data*, [Online]. [http://itrf.ensg.ign.fr/ITRF_solutions/2005/input_data.php] (Consulted in April 2008).

INTERNATIONAL TERRESTRIAL REFERENCE FRAME (2008b). *Transformation parameters: ITRS and WGS84*, [Online]. [http://itrf.ensg.ign.fr/trans_para.php] (Consulted in September 2008).

JONES, H.E. (1973). "Geodetic Datums in Canada," *Canadian Surveyor*, vol. 27, no. 3, p. 195-207.

JET PROPULSION LABORATORY (2006). *GPS Times Series*, California Institute of Technology, [Online]. [http://sideshow.jpl.nasa.gov/mbh/series.html] (Consulted in December 2006).

JET PROPULSION LABORATORY (2008). *GPS Times Series*, California Institute of Technology, [Online]. [http://sideshow.jpl.nasa.gov/mbh/series.html] (Consulted in April 2008).

LeGRAND, P., E.J.O. SCHRAMA and J. TOURNADRE (2003). "An inverse estimate of the dynamic topography of the ocean," *Geophysical Research Letters*, vol. 30, no. 2, 1062, 4 p.

LEVALLOIS, J.-J. (1969). *Géodésie générale – T1 : Méthodes générales et techniques fondamentales*, Paris, Éditions Eyrolles, 402 p.

LEVALLOIS, J.-J. (1970). *Géodésie générale – T3 : Le champ de la pesanteur*, Paris, Éditions Eyrolles, 436 p.

MIREAULT, Y., P. TÉTREAULT, F. LAHAYE, P. HÉROUX and J. KOUBA (2008). "Online Precise Point Positioning – A New, Timely Service from Natural Resources Canada," *GPS World*, vol. 19, no. 9, p. 59-64.

MEYER, T.H., D. ROMAN and D.B. ZILKOSKI (2006). "What Does *Height* Really Mean? Part III: Height Systems," *Surveying and Land Information Science*, vol. 66, no. 2, p. 149-160.

MORITZ, H. (1988). "Geodetic Reference System 1980 (GRS80)," *Bulletin Géodésique*, vol. 62, no. 3, p. 348-358.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION (2007). *NASA Oceanography*, [Online]. [http://science.hq.nasa.gov/oceans/physical/OST.html] (Consulted in January 2007).

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION (2009). *EGM96: The NASA GSFC and NIMA Joint Geopotential Model*, [Online]. [http://cddis.nasa.gov/926/egm96/egm96.html] (Consulted in October 2009).

NATIONAL GEODETIC SURVEY (1998). *National Height Modernization Study*, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Ocean Service, 181 p.

NATIONAL GEODETIC SURVEY (2008). *National Geodetic Survey – Frequently Asked Questions*, U.S. Department of Commerce, [Online]. [http://www.ngs.noaa.gov/faq.shtml] (Consulted in December 2008).

NATIONAL GEODETIC SURVEY (2009). *The National Geodetic Survey Ten-Year Plan: Mission, Vision, and Strategy – 2008-2018*, U.S. Department of Commerce, [Online]. [http://www.ngs.noaa.gov/INFO/tenyearfinal.shtml] (Consulted in July 2009).

NATIONAL IMAGERY AND MAPPING AGENCY (1997). *Department of Defense World Geodetic System 1984: Its Definition and Relationships With Local Geodetic Systems*, Technical Report TR8350.2, 3rd ed., 154 p.

PINCH, M.C. (1990). “Differences Between NAD27 and NAD83,” *Moving to NAD '83 the new address for georeferenced data in Canada*, The Canadian Institute of Surveying and Mapping, Ottawa, Ont., p. 1-15.

SCHWARZ, C.R. (1989). *The North American Datum of 1983*, Charles R. Schwarz, Editor, NOAA Professional Paper NOS 2, National Geodetic Survey, Rockville, MD, 256 p.

SOLER, T., J.D. LOVE, L.W. HALL and R.H. FOOTE (1992). “GPS Results from Statewide High Precision Networks in the United States,” *Proceedings of the 6th International Geodetic Symposium on Satellite Positioning*, The Ohio State University, Columbus, OH, 17-20 March, p. 573-582.

SOLER, T. and R.A. SNAY (2004). “Transforming positions and velocities between the International Terrestrial Reference Frame of 2000 and North American Datum of 1983,” *Journal of surveying engineering*, ASCE, vol. 130, no. 2, p. 49-55.

STEWART, R.H. (2007). *Introduction to Physical Oceanography,* Department of Oceanography, Texas A&M University, [Online]. [http://oceanworld.tamu.edu/resources/ocng_textbook/chapter10/chapter10_03.htm] (Consulted in January 2007).

VANÍČEK, P. and E.J. KRAKIWSKY (1982). *Geodesy: The Concepts*, North Holland Publishing Co., Amsterdam, 691 p.

VÉRONNEAU, M. (2006). *Geoid modeling*, Canadian Geodetic Reference System Committee, Ottawa, 18-20 September.

VÉRONNEAU, M., A. MAINVILLE and M. CRAYMER (2001). *The GPS Height Transformation (v2.0)*, Geodetic Survey Division, Natural Resources Canada, 119 p.

VÉRONNEAU, M., R. DUVAL and J. HUANG (2006). “A gravimetric geoid model as a vertical datum in Canada,” *Geomatica*, vol. 60, no. 2, p. 165-172.

ZILKOSKI, D.B., J.H. RICHARDS and G.M. YOUNG (1992). “Results of the General Adjustment of the North American Vertical Datum of 1988,” *Surveying and Land Information Systems*, American Congress on Surveying and Mapping, vol. 52, no. 3, p. 133-149.

Appendix A

ITRF2005

ALTAMIMI, Z., X. COLLILIEUX, J. LEGRAND, B. GARAYT and C. BOUCHER (2007), "ITRF2005: A new release of the International Terrestrial Reference Frame based on time series of station positions and Earth Orientation Parameters," *Journal of Geophysical Research*, 112, B09401.

Unlike the past International Terrestrial Reference Frame (ITRF) versions where global long-term solutions were combined, the ITRF2005 uses as input data time series (weekly from satellite techniques and 24-h session-wise from Very Long Baseline Interferometry) of station positions and daily Earth Orientation Parameters (EOPs). The advantage of using time series of station positions is that it allows to monitor station non-linear motion and discontinuities and to examine the temporal behavior of the frame physical parameters, namely the origin and the scale. The ITRF2005 origin is defined in such a way that it has zero translations and translation rates with respect to the Earth center of mass, averaged by the Satellite Laser Ranging (SLR) time series spanning 13 years of observations. Its scale is defined by nullifying the scale and its rate with respect to the Very Long Baseline Interferometry (VLBI) time series spanning 26 years of observations. The ITRF2005 orientation (at epoch 2000.0) and its rate are aligned to the ITRF2000 using 70 stations of high geodetic quality. The estimated level of consistency of the ITRF2005 origin (at epoch 2000.0) and its rate with respect to the ITRF2000 is respectively 0.1, 0.8, 5.8 mm and 0.2, 0.1, 1.8 mm/yr along the X, Y and Z-axis. We estimate the formal errors on these components to be 0.3 mm and 0.3 mm/yr. We believe that this low level of agreement between the two frame origins is most probably due to the poor SLR network geometry and its degradation over time. The ITRF2005 combination involving 84 co-location sites revealed a scale inconsistency of 1 ppb (6.3 mm at the equator), at epoch 2000.0, and 0.08 ppb/yr between the SLR and VLBI long-term solutions as obtained by the stacking of their respective time series. Possible causes of this inconsistency may include the poor SLR and VLBI networks and their co-locations, local tie uncertainties, systematic effects and possible inconsistent model corrections used in the data analysis of both techniques. For the first time of the ITRF history, the ITRF2005 rigorous combination provides self-consistent series of EOPs, including Polar Motion from VLBI and satellite techniques and Universal Time and Length of Day from VLBI only. A velocity field of 152 sites with an error less than 1.5 mm/yr is used to estimate absolute rotation poles of 15 tectonic plates that are consistent with the ITRF2005 frame. This new absolute plate motion model supersedes and significantly improves that of the ITRF2000 which involved six major tectonic plates.

Appendix B

Versions of NAD83 (CSRS)

Version 1

This version refers to the first ITRS — NAD83 transformation. The CACS-1 and CBN-1 solutions are based on the few CACS stations known in the ITRF92 solution and transformed to ITRF89. The CBN-1 solution comes primarily from GPS observations done in 1994. The Geodetic Survey Division gave the name NAD83 (CSRS96) to the data resulting from this transformation. The "96" in the name refers to the year this transformation was introduced, and not the reference epoch.

Version 2

This version refers to the second ITRS — NAD83 transformation. The CACS-2 and CBN-2 solutions are based on the few CACS stations known in ITRF96 (1997.0). The CBN-2 solution comes from GPS observations done from 1994 to 1997, primarily on the CBN and other geodetic points. The observation epoch used for the calculations is 1997.0.

Initially, the Geodetic Survey Division called it NAD83 (CSRS98) to differentiate it from the previous one. The "98" refers to the year this transformation was introduced, and not the reference epoch. Since the positions from the first version were almost never used, the year of introduction was removed from the name.

The Direction de la référence géodésique based the A2, A3, B and C networks on the CBN-2 solution.

Version 3

Introduced in 2000, the CACS-3 and CBN-3 solutions are based on the few known CACS stations in ITRF97 (1997.0). The CBN-3 solution comes from GPS observations used in CBN-2, to which new GPS observations were added.

A CBN-3.1 version was introduced in 2001 subsequent to new GPS observations on part of the CBN and on other geodetic points. All the calculations were done at observation epoch 1997.0.

Version 4

Introduced in 2003, the CACS-4 and CBN-4 solutions are based on the few known CACS stations in ITRF2000 (1997.0). The addition of GPS observations to versions 3 and 3.1 over the years only integrated other points into version 2. In 2001, the Geodetic Survey Division started a GPS observation campaign on all the CBN points. The campaign ended in 2002. All the calculations were done at the common observation epoch, which was 2002.0. CBN-4 is the result of this new data set.

Version 5

In 2005, the Geodetic Survey Division started a new GPS observation campaign on all the CBN points. The campaign ended in 2006. The CACS-5 and CBN-5 solutions are based on the few known CACS stations in ITRF2005 (2000.0). All the calculations were done at common observation epoch 2006.0. CBN-5 is the result of this new data set. In April 2010, the CACS-5 and CBN-5 solutions were not available yet.

Version 6

In summer 2010, the Geodetic Survey Division started a new GPS observation campaign on all CBN points. The campaign will end in 2011. The CACS-6 and CBN-6 solutions will be based on the few known CACS stations in ITRF2008 (2005.0). All the calculations will be done at observation epoch 2010.0. CBN-6 will be the result of this new data set.

Appendix C

CGVD28: Order 630 of March 11, 1935

« AT THE GOVERNMENT HOUSE AT OTTAWA

« MONDAY, the 11th day of MARCH, 1935.

« PRESENT:

« HIS EXCELLENCY

« THE GOVERNOR GENERAL IN COUNCIL:

« WHEREAS the Minister of the Interior reports that the Geodetic Service of the Department of the Interior has been carrying on precise levelling since 1906;

« That in 1931 the Department of Public Works transferred all original field books and records, resulting from precise level operations carried on by that Department since 1883, to the Geodetic Service of the Department of the Interior;

« That since that time all precise levelling in Canada has been brought under one organization and has now been consolidated into one national system of levels referred to one datum plane;

« That a number of different datum planes have been, and still are, in use in Canada and that this condition tends to confusion;

« That mean sea level has been generally adopted by other countries as their datum plane for elevations and is the datum plane to which the national system of precise levels of the Geodetic Service of the Department of the Interior has been referred;

« THEREFORE His Excellency the Governor General in Council, on the recommendation of the Minister of the Interior, is pleased to order and it is hereby ordered that mean sea level, as determined at coastal points by the Canadian Hydrographic Service and extended inland by the Canadian Geodetic Service, shall be the official datum plane for elevations in Canada and shall be known as the Canadian Geodetic Datum;

« His Excellency in Council is further pleased to order that the elevations of all works or projects of the Government of the Dominion of Canada which may originate after the date hereof shall be referred to this datum, where possible. »

Ressources naturelles
et Faune
Québec