

Ratio estimation to improve estimates of top height from suboptimal samples in forest inventory plots in Québec

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ABSTRACT

Top height is one of the most useful variables provided by sample plot measurements conducted during forest inventory programs. Combined with tree age, it allows the determination of site index, a common measure of forest site productivity. Unfortunately, there are cases where the number of site trees is suboptimal; this potential problem can lead to imprecise estimates of top height and site index. We approached this problem using ratio estimation techniques and incomplete data sets from forest inventory plots in Québec. We tested different combinations of suboptimal sampling of site trees to estimate top height based on their diameters, using only 2 or 3 site trees per 0.04-ha plot, instead of 4. The results show that suboptimal sampling of site trees should not be conducted randomly, and that some combinations consistently performed better than others. Also, some combinations never outperformed others and should be avoided when estimating top height of some important tree species in Québec using suboptimal sampling of site trees. Finally, for the best combinations, we concluded that the ratio estimation technique generally provided precise and unbiased estimates of top height.

Key words: top height, site index, site trees, ratio estimation, forest inventory

RÉSUMÉ

La hauteur dominante est une des variables les plus utiles découlant de l'échantillonnage de placettes d'inventaire forestier. Lorsqu'elle est combinée à l'âge de l'arbre, elle permet de déterminer l'indice de qualité de station (IQS), lequel représente une mesure pratique de la productivité d'une station. Malheureusement, il y a des cas où le nombre d'arbres échantillons prévus à cet effet est sous-optimal et ce problème potentiel peut engendrer des estimations imprécises de la hauteur dominante et de l'IQS. Nous avons abordé ce problème en utilisant la technique de l'estimateur ratio combinée à des données incomplètes de placettes d'inventaire forestier du Québec. Nous avons testé différentes combinaisons sous-optimales d'arbres-échantillons afin d'estimer la hauteur dominante sur la base des plus gros diamètres à hauteur de poitrine, en utilisant seulement deux ou trois arbres par placette de 0,04 hectare au lieu de quatre. Les résultats montrent qu'un échantillonnage sous-optimal ne devrait pas être réalisé au hasard et que certaines combinaisons d'arbres donnent des estimations de la hauteur dominante plus précises que d'autres. De plus, certaines combinaisons sous-optimales ont systématiquement mal estimé la hauteur dominante des placettes et ne devraient donc jamais être utilisées à cet effet. Enfin, nous concluons que la méthode d'estimation de la hauteur dominante à l'aide de l'estimateur ratio s'est avérée généralement précise et non biaisée pour les meilleures combinaisons d'arbres échantillons en mode d'échantillonnage sous-optimal.

Mots clés : hauteur dominante; indice de qualité de station; arbres indicateurs de la station; estimateur ratio; inventaire forestier



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Introduction

Stand top height (H_d) is one of the most useful variables obtained from sample plot measurement conducted during forest inventory programs. It is generally estimated using site trees, i.e. the tallest or largest diameter, free-growing, uninjured trees in a sample plot (Carmean 1975, Monserud 1984, Carmean and Lenthall 1989, Nigh and Love 1999). As a consequence, H_d is generally defined as the height of the largest-diameter tree on a 0.01-ha plot, providing the tree is suitable (e.g., Forest Productivity Council of British Columbia 1998, Sharma *et al.* 2002). For a 0.04-ha plot, this represents the height of 4 top-height trees based on the largest-diameter trees. Combined with a reference age commonly set at 50 years and taken at breast height (generally 1.30 m, Curtis and Marshall 2005), site index (SI), a common measure of forest site productivity, is determined.

Trees used to estimate SI in forest inventory plots in the province of Québec, which are generally referred to as site trees, are not always top-height trees. In many instances, site tree selection is based on criteria that are not always geared towards the determination of H_d or SI. For instance, owing to financial constraints it is sometimes impossible to measure 4 top-height trees per 0.04-ha plot. As a consequence, the number of site trees available may not be sufficient enough to adequately estimate H_d and SI. In Québec, site trees have traditionally been chosen based on their representativeness, responding to the following criteria: located close to the plot centre, a DBH (diameter at breast height) close to the average DBH of codominant and dominant trees, and belonging to the codominant or dominant canopy class. There are 3 representative site trees measured in temporary sample plots (TSP; MRNF 2005a), whereas 4 are chosen in the case of permanent sample plots (PSP; MRNF 2005b). Because in practice the number of site trees that qualify as top-height trees generally ends up being lower than 4 (optimal number) for both types of sample plots, it is necessary to estimate H_d using incomplete data.

One way to approach this problem is to use a regression equation that estimates top height using other variables, such as top diameter, mean height of site trees and mean diameter, as proposed by Bégin and Raulier (1995) and implemented in Québec by Pothier and Savard (1998, p. 15). While standard regression can be used in these circumstances, it is not the only procedure that can increase sampling efficiency by taking advantage of additional information from supplementary variables (Avery and Burkhart 2002). Alternative methods, such as ratio-of-means and means-of-ratios estimation (e.g., Ek 1971, Zarnoch and Bechtold 2000, van Hees 2002) and random coefficient models or mixed-effects models (Lynch 2005, Temesgen *et al.* 2008) have been used for forestry applications and are worth testing for H_d estimation.

Our main objective is to quantify the effect of using a suboptimal number of top-height trees (i.e., <4) among the 4 largest trees when calculating sample plot top height using

ratio estimation techniques. The general hypothesis is that a particular combination of trees among the 4 top-height trees from a 0.04-ha plot is preferable to a purely random selection. Our working hypotheses are that estimating top height with only 2 trees out of 4 H_d trees in a TSP and only 3 trees out of 4 H_d trees in a PSP can be done with reasonably low bias and variance. Using these restrictions, we more specifically want to answer the following questions: i) can an optimal choice of tree selection be made among the 4 H_d trees? ii) does the fact of selecting only 2 trees among the 4 H_d trees (TSP) or selecting only 3 trees among the 4 H_d trees (PSP) result in biases when estimating plot H_d ? iii) what is the error (variance) when only 2 trees among the 4 H_d trees (TSP) or only 3 trees among the 4 H_d trees (PSP) are selected for estimating plot top height?

Methods

Data

Data for this study come from 2 distinct sources, which allowed us to gather inventory data from a total of 68 plots. Since tree height data was available for each tree in a plot, we were able to test the use of a suboptimal number of H_d trees for the calculation of stand H_d . The first data set, thereafter named CÔTE-NORD data set, comprises a total of 36 circular plots (0.04 ha), established in 6 different black spruce stands located in the balsam fir/white birch bioclimatic domain (Fig. 1, Robitaille and Saucier 1998). The data were collected from 1997 to 1999 and come from a project initially aimed at studying the effect of harvesting with protection of small merchantable trees (Jean-Pierre Carpentier, Direction de la recherche forestière, Ministère des Ressources naturelles et de la Faune [retired], personal communication, 1999) and for estimating SI (Mailly *et al.* 2004).

The second data set, thereafter named RÉSEF data set, comprises a total of 32 stem-mapped sample plots from the RÉSEF network (Réseau d'Étude et de Surveillance des Écosystèmes Forestiers au Québec, Gagnon *et al.* 1994). These plots, of rectangular form, were established in different stands located in the province of Québec (Table 1, Fig. 1). They have been the subject of several measurements in the past, in which the DBH and height of all stems were measured 3 or 4 times. The following stand types and plot sizes were available for this study: 17 sugar maple stands, plot size of 50 m by 100 m; 7 black spruce stands, plot size of 50 m by 50 m; 6 balsam fir

Table 1. Characteristics of the sample plots

Stand type	Number of plots	Plot dimension (ha)	Density (stems ha ⁻¹) ^a	Basal area (m ² ha ⁻¹) ^a	Top height (m) ^a
CÔTE-NORD data set (Mailly <i>et al.</i> 2004)					
Black spruce	36	0.04	8843 (737)	34.1 (1.5)	13.1 (0.5)
RÉSEF data set (Gagnon <i>et al.</i> 1994)					
Black spruce	7	0.25	2613 (559)	31.2 (2.6)	17.7 (0.7)
Balsam fir	6	0.25	1487 (430)	20.3 (5.9)	16.5 (2.2)
Balsam fir	1	0.08	1838 (–)	23.1 (–)	16.0 (–)
Sugar maple	17	0.50	846 (215)	21.7 (1.3)	22.6 (0.5)
Sugar maple	1	0.15	413 (–)	35.9 (–)	26.0 (–)

^aMean (± standard error)

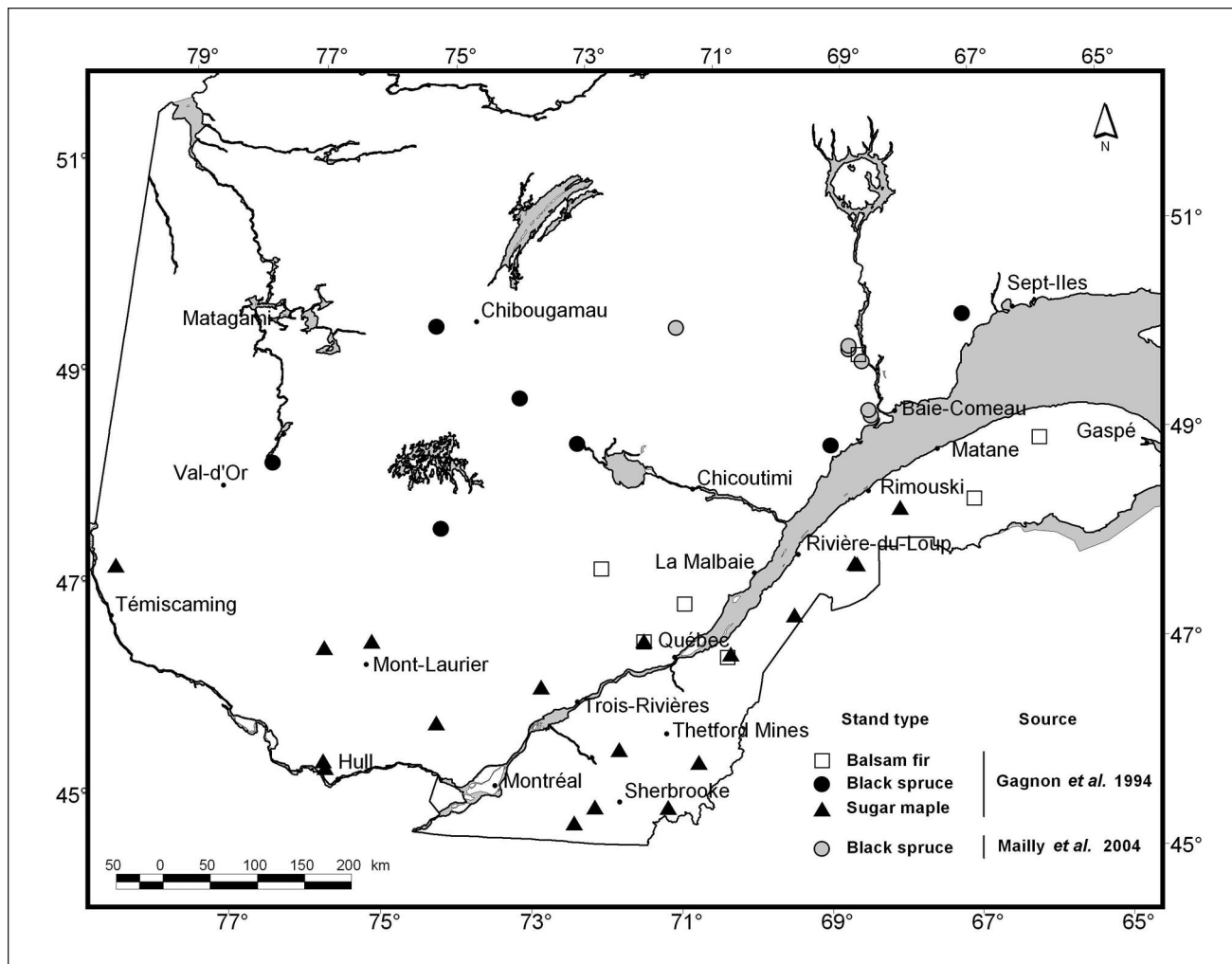


Fig. 1. Location of study plots by stand type throughout the province of Québec, Canada

stands, plot size of 50 m by 50 m; 1 balsam fir stand, plot size 40 m by 20 m; and 1 sugar maple stand, plot size of 50 m by 30 m (Table 1).

In order to answer the 3 questions asked above, all the possible combinations among the 4 larger DBH trees were used for comparison. For TSP, 6 combinations were possible, whereas 4 combinations were possible for PSP (Table 2). For each combination, the mean DBH and height of the 4 largest-DBH trees, and the mean DBH and height of the site trees selected were computed. In addition, the mean age and SI values based on the 4 largest-DBH trees and the site trees selected were computed for plots in the first data set. This was made possible because age at 1 m height of all merchantable trees was available for this data set. To avoid overestimating plot age, a tree was declared a “veteran tree” if its age happened to be twice as high as the mean age of merchantable trees in the plot.

Plot SI was calculated based on the following model for black spruce (Pothier and Savard 1998):

$$[1] SI = 0.9604 \cdot (\bar{H}_d^{0.9412}) \cdot (1 - e^{-0.03379 \cdot A}) \cdot (-0.6970 \cdot \bar{H}_d^{0.1046})$$

where $\hat{H}_d$ is top height (m) and A is the age taken at 1 m height of site trees, 1 m being the official reference height for age determination of site trees in the province of Québec.

Since inventory plots in the RÉSEF network are larger (>0.04 ha) than conventional plots, it has been necessary to simulate 0.04-ha subplots within the larger plot. A total of 2000 subplot simulations were conducted for each plot, for each measurement and for each dominant species. It was thus possible to compare mean height of H_d trees with the mean top height calculated according to the different combinations. In cases when trees in a plot were too young or when there were too few merchantable trees within a plot, the plot was withdrawn from the analysis.

Calculating the mean and variance of the top height estimator

The ratio estimation technique is a simple and convenient procedure that can be used to increase sampling efficiency by making use of information about a supplementary variable (Avery and Burkhart 2002). Since H_d trees are selected based on their diameters at breast height rather than their height, we can use the ratio estimation technique to estimate H_d (Lohr 1999, p. 61):

Table 2. List of all combinations of top-height trees as a function of the type of sample plots simulated (permanent and temporary)

Combination code	Trees selected per plot type ^a
Permanent sample plot	
P1	1, 2 and 3
P2	1, 2 and 4
P3	1, 3 and 4
P4	2, 3 and 4
Temporary sample plot	
T1	1 and 2
T2	1 and 3
T3	1 and 4
T4	2 and 3
T5	2 and 4
T6	3 and 4

^aTree number attributed by decreasing size of DBH (1 = largest DBH, 4 = smallest DBH)

$$[2] \quad \hat{H}_d = \overline{DBH}_d \frac{\sum_{i=1}^n h_i}{\sum_{i=1}^n dbh_i}$$

where $\hat{H}_d$ is the estimated top height based on 4 site trees, $\overline{DBH}_d$ is the mean DBH of the 4 largest trees of the same species (i.e. top DBH), excluding the veterans; h_i is the height of the selected tree; dbh_i is the DBH of the selected tree; n is the number of trees selected (2 for TSP or 3 for PSP). The variance of this estimator is given by the following equation (Lohr 1999, p. 68):

$$[3] \quad \hat{V}(\hat{H}_d) = \left(1 - \frac{n}{4}\right) \frac{1}{n} \frac{\sum_{i=1}^n (h_i - \hat{B} \cdot dbh_i)^2}{n-1}$$

where

$$[4] \quad \hat{B} = \frac{\sum_{i=1}^n h_i}{\sum_{i=1}^n dbh_i}$$

Since the height of all trees is known, we can compute the bias that can occur when only 2 or 3 trees are selected instead of the 4 trees normally used. Bias is calculated as follows:

$$[5] \quad Bias(\hat{H}_d) = \overline{H}_d - \hat{H}_d$$

where $\overline{H}_d$ is the height of the 4 largest trees of the same species. The root mean square error (RMSE) is calculated using the following equation:

$$[6] \quad RMSE = \sqrt{\hat{V}(\hat{H}_d) + Bias(\hat{H}_d)^2}$$

Bias and RMSE values were thus computed for each data set and for each species using equations 5 and 6, respectively. For black spruce stands of the CÔTE-NORD data set, bias and RMSE values for age estimation were also computed, using age instead of height as a variable in equations 2 to 6. RMSE values were not computed for SI since it is a stand-level variable that is estimated and not a tree-level variable. For the RÉSEF data set, bias and RMSE values for each plot were computed as the average values of the 2000 simulated 0.04-ha subplots. RMSE values were analyzed using an analysis of variance (ANOVA) procedure which takes into account the autocorrelation structure, since the site tree combinations are not independent. Means comparisons ($\alpha = 0.05$) were conducted using the MIXED procedure of SAS, version 9.1 (SAS Institute, Cary, NC). The best sampling combination for each species was chosen as the one having the lowest RMSE.

Results

Subsampling permanent sample plots

When estimating black spruce H_d using suboptimal sampling of black spruce in a permanent plot inventory program, we found that some tree combinations were more appropriate than others. For the CÔTE-NORD data set, 3 combinations including the largest-DBH tree had significantly lower RMSE values (P3 = 0.38, P2 = 0.43 and P1 = 0.48) when compared to a combination that excluded the largest-DBH tree (P4 = 0.69, Table 3). Results for black spruce with the RÉSEF data set were somewhat similar, i.e., combinations P2 and P3 had the lowest RMSE (0.61 for both), followed by combination P1 (0.64) and P4 (0.76). No differences were found when estimating age of H_d trees for black spruce of the CÔTE-NORD data set (RMSE values varying from 4.73–6.63, Table 3).

When estimating balsam fir H_d with the RÉSEF data set, the results were similar to black spruce, i.e., the 3 combinations including the largest-DBH tree had significantly lower RMSE values (P3 = 0.79, P2 = 0.79 and P1 = 0.82, respectively) when compared to combination P4 (0.94, Table 3). Results for sugar maple H_d estimation with the RÉSEF data set showed that combinations P3 and P2 had the lowest RMSE (1.60 and 1.62, respectively), followed by combination P1 (1.96) and P4 (2.82).

Subsampling temporary sample plots

Estimating black spruce H_d using suboptimal sampling of black spruce in a temporary plot inventory program also resulted in tree combinations that were more appropriate than others. In the case of the CÔTE-NORD data set, it is not possible to show the classical results of the means comparisons tests for both H_d and age because some differences were small and significant while others had higher differences but were not significantly different. In most cases, when the differences are significant the correlation between the 2 combinations is very high. The consequence in this case is that the variance of the difference is very small. Nevertheless, the results for H_d indicate that combining the largest DBH tree with the third-largest (T2), the largest-DBH tree with the fourth-largest (T3), or combining the second- and third-largest-DBH trees (T4) yielded the lowest RMSE values (0.65, 0.65 and 0.66, respectively). For age, it was found that combining the largest DBH tree with the fourth-largest (T3) or combining the second- and third-largest DBH trees (T4) yielded the lowest RMSE values (6.50, and 6.70, respectively).

Table 3. Root mean square error (RMSE) and bias values by top height tree combination for permanent sample plots

Species	Data set	Variable	N ^a	Statistic	Top height tree combination ^b			
					P1	P2	P3	P4
Black spruce	CÔTE-NORD	Top height (m)	36	RMSE	0.48a	0.43a	0.38a	0.69b
				Bias	0.22	0.04	-0.01	-0.27
		Age (years)	36	RMSE	4.73a	5.29a	5.02a	6.63a
	RÉSEF			Bias	0.17	-0.43	0.20	0.04
		Site index (m at age 50)	36	Bias	-0.05	-0.10	0.05	0.04
		Top height (m)	22	RMSE	0.64b	0.61a	0.61a	0.76c
Balsam fir	RÉSEF	Top height (m)	20	RMSE	0.82a	0.79a	0.79a	0.94b
				Bias	0.28	0.11	-0.09	-0.35
Sugar maple	RÉSEF	Top height (m)	61	RMSE	1.96b	1.62a	1.60a	2.82c
				Bias	0.91	0.47	-0.19	-1.49

^aNumber of plots^bCombinations with different letters are significantly different ($P < 0.05$)**Table 4. Root mean square error (RMSE) and bias values by top height tree combination for temporary sample plots**

Species	Data set	Variable	N ^a	Statistic	Top height tree combination ^b					
					T1	T2	T3	T4	T5	T6
Black spruce	CÔTE-NORD	Top height (m)	36	RMSE	0.80	0.65	0.65	0.66	0.70	0.90
				Bias	0.38	0.32	0.06	-0.06	-0.34	-0.43
		Age (years)	36	RMSE	7.69	7.40	6.50	6.70	7.91	8.71
	RÉSEF			Bias	-0.37	0.54	-0.35	0.31	-0.57	0.40
		Site index (m at age 50)	36	Bias	-0.23	0.00	-0.06	0.01	-0.07	0.13
		Top height (m)	22	RMSE	0.96ab	0.93a	0.99bc	0.92a	0.93a	1.02c
Balsam fir	RÉSEF	Top height (m)	20	RMSE	1.27c	1.17ab	1.21bc	1.10a	1.22bc	1.48d
				Bias	0.56	0.30	0.04	-0.06	-0.35	-0.71
Sugar maple	RÉSEF	Top height (m)	61	RMSE	3.17e	2.73b	2.91d	2.08a	2.84c	4.21f
				Bias	1.89	1.10	0.43	-0.59	-1.38	-2.77

^aNumber of plots^bCombinations with different letters are significantly different ($P < 0.05$) (lower values are better); for RMSE values, if no letters are provided this means it is not possible to show the classical results of the means comparisons tests using Proc Mixed because some differences between means were small and significant, while others had higher differences but were not significantly different.

Results for black spruce H_d with the RÉSEF data set showed that combinations T4, T2, T5 and T1 had RMSE values significantly lower (ranging from 0.92 to 0.96) than combinations T3 and T6 (0.99 and 1.02, respectively, Table 4).

When estimating balsam fir H_d with the RÉSEF data set, the results indicated that some combinations had RMSE values significantly lower ($T4 = 1.10$ and $T2 = 1.17$) than the other 4 combinations, with RMSE values ranging from 1.21 to 1.48, respectively (Table 4). In the case of sugar maple H_d with the RÉSEF data set, combination T4 ended up having a significantly lower RMSE value (2.08) when compared to the other 4 combinations, with RMSE values ranging from 2.73 to 4.21, respectively (Table 4).

Discussion and Conclusion

Results from this study indicate that some tree combinations of H_d trees consistently perform better than others for both temporary and permanent sample plot simulation exercises aimed at estimating H_d . They also show that no single combination of H_d trees systematically outperformed all others. More importantly, we found that since some combinations

never outperformed others, they should be avoided when trying to estimate H_d for some important tree species in Québec. In the case of temporary sample plots, for instance, combining only the 2 lowest-DBH trees (T6) never provides a reliable estimation of the expected plot H_d . Also, combining only the 2 largest-DBH trees (T1) or the second and the lowest-DBH trees (T5) rarely provides a reliable estimation of H_d as calculated with 4 top-height trees. For permanent sample plots, combining the 3 lowest-DBH trees (P4) never provides a reliable estimate of the expected plot H_d , whereas combining the 3 largest-DBH trees (P1) only occasionally provides a reliable estimation of plot H_d .

Interestingly, using a ratio estimator with only 2 H_d trees for black spruce yielded estimations that were somewhat accurate (i.e., both unbiased and precise, Avery and Burkhart 2002) for the best combinations in temporary sample plots. These ranged from 65 cm to 92 cm for RMSE and 6 cm to 10 cm for bias, which appear to be acceptable departures from the optimal sampling scheme with all 4 H_d trees measured. Similar figures for black spruce permanent sample plots were even better, with values that ranged from 38 cm to 61 cm for

RMSE and 1 cm to 3 cm for bias, confirming the intuitive result that H_d is better estimated using 3 trees out of 4 when compared to using only 2 trees. A similar trend was also observed for balsam fir and sugar maple, although the values for RMSE and bias were slightly higher than those for black spruce (cf. Table 3).

Using 3 trees instead of 4 per 0.04-ha plot is in fact preferable according to some authors (e.g., Matérn 1976; Rennolls 1978; Pardé and Bouchon 1988, p.131). Selecting trees in plots that are smaller than the size used for the definition of H_d (100 largest-DBH trees in a 1.0-ha plot) leads to top diameter or H_d estimates that are generally equal to or lower than those of a 1.0-ha plot (García 1998). As a consequence, it is recommended that estimates of top diameter and H_d be obtained using the n largest-DBH trees for plots of n ares (1 are = 0.01 ha = 100 m²) if $n > 50$ ares (0.5 ha) and $n - 1$ largest-DBH trees for plots of n ares if $n \leq 50$ ares (Rondeux 1999, p.187). This recommendation should be followed in order to reduce the sampling error for top diameter estimation and, to a lesser degree, H_d estimation since the height of dominant trees actually tends to stabilize at the upper end of the diameter distribution. This implies that an important error in the estimation of top diameter may have small or no consequences for the estimation of H_d (Pardé and Bouchon 1988).

In conclusion, this study offers statistical evidence that sub-optimal sampling of site trees for top height estimation should not be conducted randomly, and that certain combinations of site trees provide more precise and less biased estimates than do others. More importantly, the study shows that some combinations should clearly be avoided because they systematically yielded higher RMSE and bias values. Our results also show that sub-sampling with only 2 site trees as proposed for TSP yielded higher bias and RMSE values when compared to sub-sampling for PSP using 3 site trees. This leads us to suggest that using 3 top-height trees per plot for TSP in Québec would help reduce the variance of H_d estimates. This study has shown that the ratio estimation technique provides generally precise estimates of top height and is useful for determining the best combinations when sub-sampling a population of site trees. The technique must be used with caution, however, because ratio estimators are known to be biased for estimating means and totals (Lohr 1999). Fortunately, the bias decreases as sample size increases and the reduced variance of the ratio estimator usually compensates for the presence of bias.

Acknowledgements

We thank Dr. Catherine Périé and the others working on the RÉSEF project for their help with the RÉSEF data set. We are also grateful to Isabelle Auger for her comments and statistical advice. The comments made by 2 anonymous reviewers were helpful to improve the quality of the manuscript and are also acknowledged.

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